Objective of the course: In a broad perspective, the course should provide the students with a smooth and maximally painless transition from learning math on the undergraduate level, where rigor and proof are secondary to the use of algorithm based recipes for doing math exercises, to studying math on the upper division and graduate level, where the rigor and proving every statement made is the norm. In a narrow perspective, the course should prepare the students for the upper division courses treating Mathematical Analysis.

Due to students’ demand, the course MAA 3200 is currently offered every semester, including Summer C, annually.

Prerequisites: MAC 2313

Topics to be covered in the course include the following.

1) Logic and Language of Proof.
Logical propositions; rules of inference; basic methods of proof.

2) Sets.
Elements of naïve set theory; explaining as much as necessary and reasonable the axiomatic approach to set theory. Main constructions with sets.

3) Method/Principle of (Finite) Mathematical Induction (PMI).
Discussion (construction) of the natural numbers $\mathbb{N}$; the Least Element Property (LEP) of $\mathbb{N}$; Different versions of the PMI, and their equivalence to the LEP; PMI and finite recursion; the arithmetic operations, and the order on $\mathbb{N}$.

4) Relations between Sets; Functions; Orders on Sets.
Operations on relations (restrictions, composition, etc.); Equivalence relations; Functions: injections, surjections, bijections; Left and right inverses of functions; Quotient sets and quotient maps; Main types of orders (partial, total, dense, continuous, well orders).

5) Cardinality of Sets.
Finite and infinite sets; denumerable sets; countable and uncountable sets The theorem of Bernstein-Cantor-Schroeder; Cantor's theorem comparing the cardinalities of a set and of its power set.

6) Construction of the Main Number Systems.
Constructing rigorously the integral, and the rational numbers as quotient sets with two operations, and order induced from $\mathbb{N}$ on them; overview of the construction of real numbers using Dedekind cuts; the real line; uncountability of the set of real numbers.

7) The Classical Topology on $\mathbb{R}$.
Definition of topology on a set; the classical topology on $\mathbb{R}$, and use of sequences of real numbers to describe and work with it; the Bolzano-Weierstrass theorem; Cauchy sequences versus convergent sequences; arithmetic operations and a relation on the set of (Cauchy) sequences; the idea of the Cantor construction of the real numbers.

8) Limits and Continuity for Real Valued Functions of a Real Variable.
The epsilon-delta definition of limit and of continuity at a point; continuity of functions; the Intermediate Value Theorem; compact subsets of real numbers; the Weierstrass theorem.
Notes on the Outline

(i) The topics in items 1)-4) and 7)-8) have to be included in any version of a syllabus for a particular version of a MAA 3200 course. The topics in items 5) and 6) are highly desirable to be a part of such a course, but should be covered as time permits, and as the Instructor deems it appropriate.

(ii) No particular text book is suggested for the course. Any decent book on foundations of math, or any book providing a bridge to advanced mathematics most probably contains the topics listed in the outline, and can be used in the teaching process.