Control and Inverse Problems for Parabolic Equations on Graphs

Sergei Avdonin and Nina Avdonina, University of Alaska, Fairbanks, AK

We consider control and inverse problems for partial differential equations of the parabolic type on graphs motivated by biological models. We demonstrate that, for graphs without cycles, unknown coefficients of the equations and their right hand sides together with the topology of the graph and lengths of the edges can be recovered from the dynamical Dirichlet-to-Neumann map associated with the boundary vertices. For general graphs with cycles additional observations at the internal vertices are needed for stable identification. The corresponding null controllability results are also proved. The talk is based in part on joint work with Jonathan Bell (UMBC).

On the regularity of CR mappings between CR manifolds of hypersurface type.

Shiferaw Berhanu, Temple University, PA

We will present some results on the smoothness and analyticity of transversal CR mappings between two Levi-nondegenerate CR manifolds of hypersurface type.

Nonexistence of smooth Levi-flat hypersurface with positive normal bundle in compact Kähler manifolds of dimension $\geq 3$

Séverine Biard, Texas A&M University, TX

Among results of nonexistence of Levi-flat hypersurfaces in $\mathbb{CP}^n$, $n \geq 2$, conjectured by D. Cerveau in 1993, there are some generalizations to compact Kähler manifolds, particularly the conjecture given by Marco Brunella in 2008: there is no smooth Levi-flat hypersurface such that the normal bundle to the Levi foliation is positive along the leaves in compact Kähler manifolds of dimension $\geq 3$. In a joint work with Andrei Iordan, we obtained a positive answer to this conjecture by using $L^2$-weighted estimates for $\bar{\partial}$. I will give the idea of the proof and if time permits, I will discuss future projects around this result and links with other problems.

Analytic Hypoellipticity for Sums of Squares with a Single Symplectic Poisson-Treves Stratum

Antonio Bove, University of Bologna, Italy

We report on ongoing work on the analytic regularity of sums of squares operators when the characteristic manifold has a single symplectic stratum in its Poisson-Treves stratification. A microlocal pseudodifferential parametrix is constructed that maps micro analytic functions into themselves.
• Lower order perturbation and analytic vectors for globally analytic hypoelliptic operators in the torus
Paulo Cordaro, University of Sao Paulo, Brazil
In this talk we shall present recent results obtained in collaboration with N. Braun Rodrigues, G. Chinni and M.R. Jahnke on the following question: let \( P(x, D) \) be an analytic LPDO on the torus which is \( \epsilon > 0 \) subelliptic. Is it true that if \( P(x, D) \) is globally analytic hypoelliptic then \( P(x, D) + A(x, D) \) remains globally analytic hypoelliptic when \( A(x, D) \) is an analytic pseudodifferential on the torus with order \( < \epsilon \)? A related question is the following: if \( u \) is an analytic vector for \( P(x, D) \) then is it true that \( u \) is a Gevrey function of order \( m/\epsilon \), where \( m \) is the order of \( P(x, D) \)? We shall present some general classes of operators for which these questions have positive answer, and also relate the analysis with the properties of the abstract Green operator of \( P(x, D) \).

• Global solvability and global hypoellipticity for a class of vector fields on the torus
Paulo L. Dattori da Silva*, University of Sao Paulo at Sao Carlos/ICMC - Brazil
We present conditions on the coefficients of a class of vector fields on the torus which yield a characterization of the existence and regularity of periodic solutions. Diophantine conditions and connectedness of certain sub-level sets appear in a natural way in our results. This is a joint work with Adalberto Bergamasco (ICMC/USP) and Rafael Gonzalez (ICMC/USP).
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• On estimates and regularity for some systems of complex vector fields
Makhlouf Derridj, University of Rouen, France
In my talk, I give some results about estimates and regularity, mainly microlocal hypoellipticity, for particular involutive systems of complex vector fields. These systems were studied first by Francois Treves many years ago and after by other authors who studied some questions related to the necessary Treves condition for hypoellipticity, particularly H. Maire

• Exact boundary control for 1-D wave and Schrodinger equations with strong potential singularities
Julian Edward, Florida International University, FL
We consider the problem of boundary control for a one dimensional wave equation with \( N \) interior point masses. We assume the control is at the left end, and the string is fixed the right end. Singularities in waves are “smoothed” out to one order as they cross a point mass. We show that the reachable set for a \( L^2 \) control equals \((L^2 \times H^{-1}) \oplus (H^1 \times L^2) \oplus ... \oplus (H^N \times H^{N-1})\) plus some compatibility conditions. The proof reduces the control problem to a moment problem, which is then solved using the theory of exponential divided differences in tandem with a unique shape controllability result. The methods are then extended to Schrodinger-wave type equations with strong potential singularities. This is work done in collaboration with
Sergei Avdonin.

- Ill-posedness for a family of nonlinear and nonlocal evolution equations
  
  Alex Himonas, University of Notre Dame, IN
  
  We shall discuss the well-posedness of a family of nonlinear and nonlocal evolution equations depending on a parameter \( b \), which has been introduced by Holm and Staley and which is expressing a balance between evolution, convection and stretching. This family of equations possess peakon and multipeakon traveling wave solutions for all values of the parameter \( b \). Furthermore, it is well-posed for initial data in Sobolev spaces \( H^s \) for all \( s > 3/2 \). However, it has been shown recently that for \( s < 3/2 \) it is ill-posed. This work establishes \( 3/2 \) as the critical index of well-posedness in Sobolev spaces. The talk is based on work with Katelyn Grayshan, Curtis Holliman, Carlos Kenig and Gerard Misiolek.

- On the FBI transforms and their use in (microlocal) analysis
  
  Gustavo Hoepfner, Federal University at Sao Carlos, Brazil
  
  We shall consider a larger class of FBI transforms: those introduce by Berhanu and Hounie (the real part of the Hessian of the phase function can degenerate at the point of interest), and show that it can be used to characterize the classes of ultradifferentiable functions, also know as Denjoy-Carleman classes, (which contain the Gevrey classes). Applications are given, in particular we show that the FBI introduced by Christ, can be viewed as an example of the BH-FBI. This is a joint work with Renan Medrado.

- \( L^1 \) Estimates for vector valued elliptic operators
  
  Jorge Hounie, Federal University at Sao Carlos, Brazil
  
  We will describe the following result: if \( A(x,D) \) is a linear differential operator of order \( \nu \) with smooth complex coefficients in \( \Omega \subset \mathbb{R}^N \) from a complex vector space \( E \) to a complex vector space \( F \), the Sobolev a priori estimate
  
  \[
  \|u\|_{W^{\nu-1,N/(N-1)}} \leq C \|A(x,D)u\|_{L^1}
  \]
  
  holds locally at any point \( x_0 \in \Omega \) if and only if \( A(x,D) \) is elliptic and the constant coefficient homogeneous operator \( A_{\nu}(x_0,D) \) is canceling in the sense of Van Schaftingen for every \( x_0 \in \Omega \) which means that
  
  \[
  \bigcap_{\xi \in \mathbb{R}^N \setminus \{0\}} a_{\nu}(x_0,\xi)[E] = \{0\}.
  \]
  
  Here \( A_{\nu}(x,D) \) is the homogeneous part of order \( \nu \) of \( A(x,D) \) and \( a_{\nu}(x,\xi) \) is the principal symbol of \( A(x,D) \). This result implies and unifies the proofs of several estimates for complexes and pseudo-complexes of operators of order one or higher proved recently by other methods as well as it extends in the local setup the characterization of Van Schaftingen to operators with variable coefficients. This is joint work with T. Picon.
Local measure preserving maps between hermitian symmetric spaces of compact type.
Xiaojun Huang, Rutgers University, NJ

Left invariant CR and pseudo-hermitian structures on $S^3$
Howard Jacobowitz, Rutgers University, Camden, NJ
The CR and pseudo-hermitian structures on $S^3$ invariant under the action of $SU(2)$ are classified using the moving frames method of Cartan and Webster, respectively. Let

$$L = \overline{w} \frac{\partial}{\partial z} - \overline{z} \frac{\partial}{\partial w}$$
and $\theta = -i(zdz + \overline{w}dw)$

and let $\mu$ be a complex constant different from $\pm 1$. Then $L + \mu \overline{L}$ defines all left invariant CR structures on $S^3$ with the CR distribution $\theta^\perp$ (except for that corresponding to $\overline{L}$).

**Theorem.** CR structures given by $\mu_1$ and $\mu_2$ are equivalent if and only if either $|\mu_1| = |\mu_2|$ or $|\mu_1| = |\mu_2|^{-1}$. If $|\mu_1| = |\mu_2|$ the CR equivalence can be chosen to preserve $\theta$ and so the two structures are also equivalent in the pseudo-hermitian sense.

Some results about invariance under the action of other three dimensional groups will be discussed. If time permits, conjugate CR structures will be introduced.

Uniqueness of functions for the Selberg class
Haseo Ki, Yonsei University, Korea
Selberg axiomatized properties of $L$-functions and defined the Selberg class which would coincide with the class of all arithmetically meaningful $L$-functions. Also, we can define the extended Selberg class. I will introduce uniqueness theorems for functions in these classes.

On the equivalence of real-analytic Cauchy-Riemann manifolds
Ilya Kossovskiy, University of Vienna, Austria
Interplay between different types of equivalence in CR-geometry has attracted considerable attention in the last 20 years. Usually one considers the following 3 types of equivalence for CR-manifolds: biholomorphic equivalence, smooth CR equivalence, and formal equivalence. Using our recently developed theory on connecting degenerate CR-manifolds and Dynamical Systems, we showed that these 3 sorts of equivalence are significantly different from each other in for the case of infinite type CR-manifolds. In connection with that, I will concern in this lecture our recent result with Lamel and Stolovitch showing that for infinite type hypersurfaces in $\mathbb{C}^2$ their equivalence in the formal category implies that in the smooth category.

Efficient construction of transmutations and a new representation for solutions of Sturm-Liouville equations, uniform with respect to the spectral parameter
Vladislav Kravchenko, Cinvestav Queretario, Mexico
Let $q \in C[-b, b]$ be a complex valued function. Consider the Sturm-Liouville equation

$$Ay = y'' - q(x)y = -\omega^2 y.$$  
(1)

It is well known (see, e.g., [2]) that there exists a Volterra integral operator $T$ called the transmutation (or transformation) operator defined on $C[-b, b]$ by the formula

$$Tu(x) = u(x) + \int_{-x}^{x} K(x, t)u(t)dt$$

such that for any $u \in C^2[-b, b]$ the following equality is valid

$$ATu = Tu''$$

and hence any solution of (1) can be written as $y = T[u]$ where $u(x) = c_1 \cos \omega x + c_2 \sin \omega x$ with $c_1$ and $c_2$ being arbitrary constants.

The transmutation kernel $K$ is a solution of a certain Goursat problem for the hyperbolic equation

$$\left( \frac{\partial^2}{\partial x^2} - q(x, t) \right) K(x, t) = \frac{\partial^2}{\partial t^2} K(x, t)$$

In the talk I present an exact representation for $K$ in the form of a Fourier-Legendre series with explicit formulas for the coefficients. As a corollary of this result, a new representation of solutions to equation (1) is obtained. For every $x$ the solution is represented as a Neumann series of Bessel functions depending on the spectral parameter $\omega$. Due to the fact that the representation is obtained using the corresponding transmutation operator, a partial sum of the series approximates the solution uniformly with respect to $\omega$ which makes it especially convenient for the approximate solution of spectral problems. The numerical method based on the proposed approach allows one to compute large sets of eigendata with a nondeteriorating accuracy. The talk is based on [1].

References

**A necessary inequality for the Schrödinger equation**
Mark Leckband, Florida International University, FL

The existence of a nontrivial solution $u$ to the Schrödinger equation $-\Delta u = Vu$, $V$ not identically 0 on a domain $D$ requires the potential $V$ satisfy an inequality of the form $K||V|| \geq 1$. Such inequalities for a bounded domain $D$ are derived in recent work by De Carli, Edward, Hudson, and Leckband.

We discuss the derivation of the inequalities on $V$ that follow from Dirichlet- or Neumann boundary conditions. The inequalities involve the product of powers of an appropriate norm of $V$, the measure of $D$, and a sharp
Sobolev constant. These results generalize to the equation with the $p$-Laplacian, $-\Delta_p u = V u |u|^{p-2}$, for $1 < p < \infty$. In most cases the inequalities are best possible.

- **Estimates on zeros of Dirichlet series**  
  Bao Qi Li, Florida International University, FL  
  We consider lower bounds on the number of zeros of a Dirichlet series and give an affirmative answer to an open problem of Bombieri and Perelli on the bound. An application is given to estimate the cardinality of the symmetric difference for zeros of L-functions.

- **The classification problem for arclength null-quadrature domains**  
  Erik Lundberg, Florida Atlantic University, FL  
  A null-quadrature domain is a (necessarily unbounded) domain over which integration of any function in the Bergman space vanishes. M Sakai classified (planar) null-quadrature domains in 1981. Considering integration over the boundary of the domain leads to the notion of an arclength null-quadrature domain. The corresponding classification problem remains open. We discuss recent progress and useful unexpected connections to fluid dynamics and minimal surfaces. This includes joint work with Dmitry Khavinson and Razvan Teodorescu and also joint work with Alexandre Eremenko.

- **Heat Content and Dirichlet Spectrum**  
  Patrick McDonald, New College of Florida, FL  
  It is well known that Dirichlet spectrum is a sequence of isometry invariants deeply related to the geometry of the domain upon which the underlying Laplace operator is defined. There are a great number of forward and inverse results, with application across a broad range of disciplines. In this talk I will discuss a second sequence of geometric invariants defined using the Laplace operator. These invariants first arose in the study of elastic bodies, and were later shown to be closely related to the heat content of the domain upon which the Laplace operator is defined. In this talk I will introduce the required definitions and machinery, survey the key background results and analogs of results for Dirichlet spectrum, and discuss recent results involving how heat content can be used to extract information about Dirichlet spectrum and domain geometry.

- **Friedrichs extension of semibounded elliptic operators on a manifold with singularities**  
  Gerardo Mendoza, Temple University, PA  
  The manifold with singularities under consideration is modeled as a smooth compact manifold $\mathcal{M}$ whose boundary is the total space of a fibration $\mathcal{N} \to \mathcal{Y}$ with compact fibers. This generalizes the situation resulting from introducing cylindrical coordinates on a smooth manifold along a submanifold $\mathcal{Y}$. The operators to be discussed have the structure of an operator written in cylindrical coordinates; there is a notion of ellipticity for such operators. Let $E \to \mathcal{M}$ be a Hermitian vector bundle and $A$ one such
operator,

\[(\dag) \quad A : C_c^\infty(\mathcal{M}; E) \subset x^{-\nu}L^2_b(\mathcal{M}; E) \to x^{-\nu}L^2_b(\mathcal{M}; E),\]

of second order, elliptic and semibounded. The space \(x^{-\nu}L^2_b(\mathcal{M}; E)\) is defined using the Hermitian metric of \(E\), a defining function \(x\) for the boundary and a density \(x^{-1}m\) with \(m\) smooth and positive. I will describe the domain of the Friedrichs extension of (\dag) under certain mild natural conditions. This is joint work with T. Krainer.

- **Integral inequalities that imply smoothness**

Mario Milman, Instituto Argentino de Matematica, Argentina, and Florida International University, FL

I will discuss the latest results (cf. [5]) of a project (cf. [2], [3], [4]) to extend the scope of inequalities obtained by A. Garsia and his collaborators in the late sixties and seventies (cf. [1], [2], and the references therein). The Garsia inequalities have had many interesting applications. In this talk I will discuss new integral inequalities of Garsia type that imply smoothness and their connection with isoperimetry. New applications will be provided.

**References**


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- **Artin approximation and CR geometry**

Nordine Mir, Texas A&M at Qatar

In 1968, Artin proved his famous approximation theorem: given any system of real-analytic equations, if there exists a formal solution to such a system at a given point, then there exists a real-analytic solution that is as close as we want in the Krull topology to the formal solution. One question that naturally thereafter arises is whether the conclusion of Artin’s approximation theorem is still preserved if the system of equations is coupled with a specific PDE. In 1978, Milman investigated such a question when the PDE consists of the standard Cauchy-Riemann operator in \(\mathbb{R}^{2n} \simeq \mathbb{C}^n\): he showed that any formal solution of a system of real-analytic equations and of the standard CR equations in \(\mathbb{C}^n\) can be approximated (in the Krull topology) by a sequence of convergent solutions of the system of analytic and CR equations. In this talk, we will discuss recent results generalizing Milman’s theorem when the standard Cauchy-Riemann operator in \(\mathbb{C}^n\) is
replaced by the tangential Cauchy-Riemann operator associated to a real-analytic CR manifold.

- **The art of integration by parts**  
  *Irina Mitrea, Temple University, PA*  
  The Integration by Parts Formula, which is equivalent with the Divergence Theorem, is one of the most basic tools in Analysis. Originating in the works of Gauss, Ostrogradsky, and Stokes, the search for an optimal version of this fundamental result continues through this day and these efforts have been the driving force in shaping up entire subbranches of mathematics, like Geometric Measure Theory.

  In this talk I will review some of these developments (starting from elementary considerations to more sophisticated versions) and I will discuss recent results regarding a sharp divergence theorem with non-tangential traces. This is joint work with Dorina Mitrea and Marius Mitrea from University of Missouri, Columbia.

- **A Harnack Inequality for semi-linear equations in some Nilpotent Lie groups**  
  *Ahmed Mohammed, Ball State University, IN*  
  We will discuss Harnack inequality for positive solutions of semi-linear sub-Laplacian equations in bounded domains G of Carnot groups of Heisenberg type. The existence and asymptotic boundary estimates of solutions of semi-linear equations that take infinite value on the boundary of G will be used to obtain the Harnack inequality.

- **Big denominators and analytic normal forms.**  
  *Laurent Stolovitch, University of Nice, France*  
  We study the regular action of an analytic pseudo-group of transformations on the space of germs of various analytic objects of local analysis and local differential geometry. We fix a homogeneous object $F_0$ and we are interested in an analytic normal form for the whole affine space $\{F_0 + h.o.t.\}$. We prove that if the cohomological operator defined by $F_0$ has the big denominators property and if a formal normal form is well chosen then this formal normal form holds in analytic category. We also define big denominators in systems of nonlinear PDEs and prove a theorem on local analytic solvability of systems of nonlinear PDEs with big denominators. Moreover, we prove that if the denominators grow “relatively fast”, but not fast enough to satisfy the big denominator property, then we have a normal form, respectively local solvability of PDEs, in a formal Gevrey category. We illustrate our theorems by explanation of known results and by new results in the problems of local classification of singularities of vector fields, non-isolated singularities of functions, tuples of germs of vector fields, local Riemannian metrics and conformal structures.

- **Global stratification of analytic varieties, symplectic and Poisson partitions**  
  *François Treves, Rutgers University, NJ*
The lecture describes a simple construction of a global analytic stratification of an analytic subvariety $V$ of a $C^\infty$ manifold $M$. The construction exploits the local stratifications introduced by S. Łojasiewicz in the late 1950’s. It produces the real analogue of the natural stratification in the complex case. Under the assumption that $M$ is a symplectic manifold the stratification can be “refined” to provide a partition of $V$ into countably many $C^\infty$ submanifolds $L_j$ with the property that the pullback to each of them of the fundamental symplectic two-form has constant rank. Given a system of functions $f_1, \ldots, f_r$ in $M$ and the Lie algebra spanned by the Poisson multibrackets $f_I = \{f_N, \{f_{N-1}, \ldots \{f_1, f_{m}\} \cdots \}$ we refine the symplectic partition to decompose $V$ as a union of countably many pairwise disjoint $C^\infty$ submanifolds $N_k$ on which not only is the rank of the pullback of $\varpi$ to $N_k$ constant but so is also the type function $x \mapsto \tau(x)$, the smallest length $|I|$ of a multibracket $f_I(x) \neq 0$.

- **Bergman-Weil expansion for holomorphic functions**
  *Alecos Vidras, University of Cyprus, Cyprus*
  Let $U \subset \mathbb{C}$ be a bounded domain with piecewise smooth boundary. Motivated by ideas coming from Several Complex Variables (in particular from the multiplicative theory of residue currents in the non complete intersection case as well as from weighted Cauchy or Cauchy-Pompeiu formulae), we state division interpolation formulae of the Lagrange type with respect to the powers of an ideal $(f_1, \cdots, f_m)$ (when $m > 1$) and derive from them a “balanced” Bergman-Weil type convergent expansion of $h$ in the domain $\{z \in U : ||f(z)|| < \min_{\mathbb{C}^U} ||f(\zeta)||\}$ in terms of $f_k^2$, $k \in \mathbb{N}^m$. Our approach is based on complex duality ideas and multidimensional residue theory, adapted here to the one variable setting. (Joint work with A.Yger)

- **Dynamics of Newton Map and Complexity**
  *Yuefei Wang, Chinese Academy of Sciences, China*
  We will talk about the dynamics of Newton map and geometry of polynomials. Problems and recent results on complexity of polynomials, rigidity of stable algebraic families of rational maps and iterative algorithms will be discussed.

- **2D magnetohydrodynamic (MHD) equations with partial dissipation**
  *Jiahong Wu, Oklahoma State University, OK*
  The magnetohydrodynamic (MHD) equations model electrically conducting fluids in the presence of a magnetic field such as plasmas and liquid metals. They are a combination of the Navier-Stokes equations (with Lorentz force) and the electromagnetic equations. The global (in time) regularity problem concerning the MHD equations have garnered considerable interest recently. This talk focuses on recent developments on the 2D MHD equations with partial dissipation. When there is only partial dissipation, the global regularity problem can be extremely difficult. We report very recent global regularity results for two partial dissipation cases: the MHD
equations with no magnetic diffusion and the MHD equations with no velocity dissipation. Some of the results are joint work with Chongsheng Cao.

- **Holomorphic isometries from the unit ball into the irreducible classical bounded symmetric domain**
  
  *Ming Xiao, Rutgers University, NJ*
  
  We will talk about local holomorphic isometries from the unit ball into the irreducible classical bounded symmetric domain, in particular of type IV. We will present a classification result of such holomorphic isometries up to automorphisms in the maximal dimensional case. We also discuss some phenomena of such mappings in contrast with proper mappings between balls. In particular, we will give an answer to a question regarding boundary singularities raised by Mok. It is a joint work with Y. Yuan.

- **Expectation of the locations of Gaussian holomorphic functions’ zero-set**
  
  *Zhuan Ye, Northern Illinois University, IL*
  
  Let $G_n = \sum a_\alpha f_1^{\alpha_1} f_2^{\alpha_2} \cdots f_\ell^{\alpha_\ell}$ be a polynomial in terms of holomorphic functions $f_1, f_2, \cdots, f_\ell$ from a region $\Omega \subset \mathbb{C}^m$ to $\mathbb{C}$, where $\alpha = (\alpha_1, \cdots, \alpha_\ell)$ is an index and $a_\alpha$ are independent complex-valued random variables defined on a probability space with standard Gaussian distribution. We find the limit of the sequence of the expectation of the normalized zero current of $G_n(z)$ in the sense of currents and an expression of the limit. With this expression, we can easily obtain some known results and new corollaries.