1. Amy (A), Betsy (B), Carla (C), Doris (D), and Emilia (E) are candidates for an open Student Government seat. There are 110 voters with the preference lists below.

<table>
<thead>
<tr>
<th></th>
<th>36</th>
<th>24</th>
<th>20</th>
<th>18</th>
<th>8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>E</td>
<td>D</td>
<td>B</td>
<td>C</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>E</td>
<td>D</td>
<td>E</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>B</td>
<td>C</td>
<td>B</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>B</td>
<td>E</td>
<td>D</td>
<td>E</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

Who wins the election if the method used is:

a) plurality?

b) plurality with runoff?

c) sequential pairwise voting with agenda ABEDC?

d) Hare system?

e) Borda count?

2. What is the minimum number of votes needed for a majority if the number of votes cast is:

a) 120?

b) 141?

3. Consider the following preference lists:

1 1 1
A C B
B A D
D B C
C D A

If sequential pairwise voting and the agenda BACD is used, then D wins the election. Suppose the middle voter changes his mind and reverses his ranking of A and B. If the other two voters have unchanged preferences, B now wins using the same agenda BACD. This example shows that sequential pairwise voting fails to satisfy what desirable property of a voting system?

4. Consider the following preference lists held by 5 voters:

2 2 1
A B C
C C B
B A A

First, note that if the plurality with runoff method is used, B wins. Next, note that C defeats both A and B in head-to-head matchups. This example shows that the plurality with runoff method fails to satisfy which desirable property of a voting system?

5. If an election has a Condorcet winner, under which voting method is the Condorcet winner always guaranteed to finish first?
6. Ten board members vote by approval voting on eight candidates for new positions on their board as indicated in the following table. An X indicates an approval vote. For example, Voter 1, in the first column, approves of candidates A, D, E, F, and G, and disapproves of B, C, and H.

<table>
<thead>
<tr>
<th>Candidate</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>X X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>X X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>X</td>
<td>X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>X</td>
<td>X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>X</td>
<td>X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>X</td>
<td>X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>X X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>X</td>
<td>X X X X X X X X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Which candidate is chosen for the board if just one of them is to be elected?
b) Which candidates are elected if 70% approval is necessary and at most 3 are elected?

7. Multiple Choice: Consider the following preference lists:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>A</td>
<td>1</td>
</tr>
</tbody>
</table>

If the Hare system is used, A wins the election.

Now suppose the voter on the far right changes his mind and moves A above C on his preference list.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>A</td>
<td>1</td>
</tr>
</tbody>
</table>

A new vote is held, again using the Hare system, but this time B wins.

This example shows that the Hare system fails to satisfy which desirable property of a voting system?

8. Using the preference lists below and sequential pairwise voting with the agenda MAHT, T wins the election.

<table>
<thead>
<tr>
<th>8</th>
<th>9</th>
<th>6</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>H</td>
<td>A</td>
<td>H</td>
</tr>
<tr>
<td>A</td>
<td>A</td>
<td>T</td>
<td>M</td>
</tr>
<tr>
<td>T</td>
<td>M</td>
<td>H</td>
<td>A</td>
</tr>
<tr>
<td>H</td>
<td>T</td>
<td>M</td>
<td>T</td>
</tr>
</tbody>
</table>

This shows that sequential pairwise voting fails to satisfy which desirable property of a voting system?
9. Consider the following preference lists in an election with 14 voters and four candidates A, B, C and D:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>A</td>
<td>B</td>
</tr>
</tbody>
</table>

a) If the Hare system is used, in what order are the candidates eliminated?
b) If a Borda count is used, how many Borda points does candidate B receive?
c) Which candidate, if any, is the Condorcet winner?

Problems 10-15 are multiple choice.

10. Which ONE of the following explains why the Borda count satisfies monotonicity?
A) The Borda count satisfies monotonicity because mono means one and each alternative will end up with exactly one total after adding up all the Borda points.
B) If everyone prefers B to D, then B receives more points from each list than D. Thus, B receives a higher total than D and so D is certainly not among the winners.
C) Suppose A wins an election and a voter changes his preference moving A above B. Since A’s Borda count increases, B’s Borda count decreases and all other Borda counts remain unchanged, A still has the most Borda points. So A still wins.
D) The Borda count satisfies monotonicity because the only way A can go from losing one election to being among the winners of a new election is for at least one voter to reverse his or her ranking of A and the previous winner.
E) This is a trick question. The Borda count does not satisfy monotonicity.

11. Which one of the following explains why sequential pairwise voting satisfies the Condorcet winner criterion?
A) Assume alternative A is the Condorcet winner. Then A is ranked at the top of every preference list. So A will win a sequential pairwise vote regardless of agenda.
B) Suppose A has a majority of the first place votes. Then A will have a majority in a head-to-head match up against all other alternatives. Since A wins all head-to-head match ups, A wins the election.
C) Suppose candidate A is the winner under sequential pairwise voting and a second vote is held in which the only change is that one voter puts A above B on his preference list. The only head-to-head pairing affected is A vs. B. If this pairing occurs in the agenda, A still wins.
D) Assume alternative A is the Condorcet winner. Then A beats every other alternative in a pairwise comparison. So A will win a sequential pairwise vote regardless of agenda.
E) Suppose alternative A is the winner under sequential pairwise voting. Suppose the only change is that one voter puts A above B. The only head-to-head pairing affected is A vs. B. If this pairing occurs in the agenda, A still wins.
12. Which one of the following explains why the Hare system satisfies the Pareto condition?
A) Suppose everyone prefers A to B. Since B cannot have any first place votes, B cannot make it into a runoff. Hence, B doesn’t win.
B) Suppose A has a majority of the first place votes. Then A can never have the fewest first place votes. Hence, A is never eliminated and wins.
C) Suppose everyone prefers A to B. Then A’s Borda count will be greater than B’s. Hence, B doesn’t win.
D) Suppose everyone prefers A to B. B cannot have a plurality since it cannot have any first place votes. Hence, B doesn’t win.
E) Suppose everyone prefers A to B. Since B cannot have any first place votes, B is eliminated in the first stage. Hence, B doesn’t win.

13. Which one of the following explains why plurality voting satisfies Monotonicity?
A) Suppose alternative A is the winner under sequential pairwise voting. Suppose the only change is that one voter puts A above B. The only head-to-head pairing affected is A vs. B. If this pairing occurs in the agenda, A still wins.
B) Suppose everyone prefers A to B. B cannot have a plurality since it cannot have any first place votes. Hence, B doesn’t win.
C) Suppose A has a majority of the first place votes. Then A has the most first place votes and is the winner.
D) Assume alternative A has the most first place votes. Any change that improves A’s ranking cannot subtract first place votes from A or add first place votes to another alternative. Hence A still has a plurality.
E) Suppose alternative A is the winner under a Borda count. Suppose the only change is that one voter puts A above B. This will increase A’s Borda count and decrease B’s. All other alternatives have unchanged counts. So A still wins.

14. Which one of the following explains why the plurality with runoff method satisfies the Pareto condition?
A) Suppose everyone prefers A to B. Since B cannot have any first place votes, B cannot make it into a runoff. Hence, B doesn’t win.
B) Suppose A has a majority of the first place votes. Then A wins without a runoff.
C) Suppose everyone prefers A to B. B cannot have a plurality since it cannot have any first place votes. Hence, B doesn’t win.
D) Suppose everyone prefers A to B. Since B cannot have any first place votes, B is eliminated in the first stage. Hence, B doesn’t win.
E) Suppose everyone prefers A to B. Then A’s Borda count will be greater than B’s. Hence, B doesn’t win.

15. Which of the following are true as a result of Arrow’s Impossibility Theorem?
A) No voting system satisfies all the desirable properties of a voting system.
B) There is not now, nor will there ever be, a perfect voting system.
C) Any voting system will have at least one flaw.
D) All are true
E) None are true
CHAPTER 11

16. Fill in the blank.
   a) A voter with no power is called a __________.
   b) A voter whose weight is greater than or equal to the quota is called a __________.
   c) A voter who constitutes a one-person blocking coalition is said to have _______ _______.

17. In the weighted voting system [7: 3, 3, 2], the number of votes needed to block is ____.

18. In the weighted voting system [8: 4, 3, 2, 1], who is the pivotal voter in the Shapley-Shubik coalition DBCA?

19. A weighted voting system with 4 voters has the following winning coalitions:
   {A, D}, {A, C}, {B, C, D}, {A, B, C}, {A, B, D}, {A, C, D}, {A, B, C, D}
   List all minimal winning coalitions.

20. Find the Shapley-Shubik power index of
   a) [7: 5, 2, 2]       b) [7: 4, 4, 2]       c) [20: 15, 1, 1, 1, 1, 1]

21. Find the Banzhaf power index of
   a) [10: 7, 5, 3]      b) [7: 4, 4, 2]

Problems 22-27 are multiple choice.

22. Which one of the following weighted voting systems is a dictatorship?
   A) [6: 2, 2, 2]       B) [6: 5, 1, 0]       C) [6: 7, 2, 1]       D) [6: 3, 3, 1]       E) none of these

23. The weighted voting system [5: 3, 3, 2] is an example of which type of voting system?
   A) unanimity       B) majority rules       C) dictatorship       D) clique       E) chair veto

24. The weighted voting system [8: 5, 3, 2] is an example of which type of voting system?
   A) unanimity       B) majority rules       C) dictatorship       D) clique       E) chair veto

25. The weighted voting system [8: 3, 3, 2] is an example of which type of voting system?
   A) unanimity       B) majority rules       C) dictatorship       D) clique       E) chair veto

26. The weighted voting system [6: 4, 3, 2] is an example of which type of voting system?
   A) unanimity       B) majority rules       C) dictatorship       D) clique       E) chair veto

27. A committee consists of three faculty members and the dean. To pass a measure, at least two faculty members and the dean must vote “yes.” Which of the following weighted voting systems describes this committee?
   A) [4: 2, 1, 1, 1]     B) [3: 2, 1, 1, 1]     C) [4: 2, 1, 1]     D) [4: 3, 1, 1, 1]     E) [5: 2, 1, 1, 1]
CHAPTER 13

28. The 1991 divorce of Donald and Ivana Trump involved 5 marital assets: a Connecticut estate, a Palm Beach mansion, a Trump Plaza apartment, a Trump Tower triplex, and cash/jewelry. Each distributes 100 points over the items in a way that reflects their relative worth to that party.

<table>
<thead>
<tr>
<th>Marital asset</th>
<th>Donald's points</th>
<th>Ivana's points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connecticut estate</td>
<td>10</td>
<td>38</td>
</tr>
<tr>
<td>Palm Beach mansion</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Trump Plaza apartment</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Trump Tower triplex</td>
<td>38</td>
<td>10</td>
</tr>
<tr>
<td>Cash and jewelry</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Use the adjusted winner procedure to determine a fair allocation of the assets.

29. After having been roommates for four years at college, Alex and Jose are moving on. Several items they have accumulated belong jointly to the pair, but know must be divided between the two. They assign points to the items as follows:

<table>
<thead>
<tr>
<th>Object</th>
<th>Alex’s points</th>
<th>Jose’s points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Textbooks</td>
<td>20</td>
<td>16</td>
</tr>
<tr>
<td>Barbells</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Rowing machine</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>Music Collection</td>
<td>8</td>
<td>11</td>
</tr>
<tr>
<td>Computer</td>
<td>15</td>
<td>17</td>
</tr>
<tr>
<td>Desk</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Cindy Margolis photos</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

Use the adjusted winner procedure to determine a fair division of the property.

30. A parent leaves a house, a farm, and a piece of property to be divided among four children who submit dollar bids on these objects as follows:

<table>
<thead>
<tr>
<th>Item</th>
<th>John</th>
<th>Paul</th>
<th>George</th>
<th>Ringo</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>180,000</td>
<td>225,000</td>
<td>250,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Car</td>
<td>12,000</td>
<td>9,000</td>
<td>10,000</td>
<td>16,000</td>
</tr>
<tr>
<td>Boat</td>
<td>26,000</td>
<td>20,000</td>
<td>24,000</td>
<td>22,000</td>
</tr>
</tbody>
</table>

What is the fair division arrived at by the Knaster inheritance procedure?

31. Use the Knaster Inheritance Procedure to describe a fair division of a house, a car, and jewelry among three heirs, A, B, and C. The heirs submit sealed bids (in dollars) on these objects as follows:

<table>
<thead>
<tr>
<th>Item</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>150,000</td>
<td>120,000</td>
<td>151,000</td>
</tr>
<tr>
<td>Car</td>
<td>20,000</td>
<td>22,000</td>
<td>10,000</td>
</tr>
<tr>
<td>Jewelry</td>
<td>10,000</td>
<td>8,000</td>
<td>4,000</td>
</tr>
</tbody>
</table>
32. Suppose we have two items (X and Y) that must be divided by Javier and Mary. Assume that Javier and Mary each spread 100 points over the items (as in the Adjusted Winner Procedure) to indicate the relative worth of each item to that person:

<table>
<thead>
<tr>
<th></th>
<th>Mary</th>
<th>Javier</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>Y</td>
<td>50</td>
<td>60</td>
</tr>
</tbody>
</table>

a) If Mary gets X and Y and Javier gets nothing, is this allocation Pareto-optimal?
If Mary gets Y and Javier gets X, is this allocation:
b) proportional?  c) envy-free?  d) equitable  e) Pareto-optimal?

33. Suppose we have four items (W, X, Y, and Z) and four people (Ralph, Alice, Ed, and Trixie). Assume that each of the people spreads 100 points over the items (as in the Adjusted Winner Procedure) to indicate the relative worth of each item to that person:

<table>
<thead>
<tr>
<th></th>
<th>Ralph</th>
<th>Alice</th>
<th>Ed</th>
<th>Trixie</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>21</td>
<td>28</td>
<td>76</td>
<td>24</td>
</tr>
<tr>
<td>X</td>
<td>24</td>
<td>23</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>Y</td>
<td>25</td>
<td>25</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>Z</td>
<td>30</td>
<td>24</td>
<td>0</td>
<td>26</td>
</tr>
</tbody>
</table>

Suppose Ralph gets Z, Alice gets Y, Ed gets W, and Trixie gets X.

a) Is this allocation proportional?
b) Who does Alice envy?
c) Is there an allocation that makes Alice better off without making anyone else worse off?
d) Find an equitable allocation. Ralph gets ___, Alice gets ___, Ed gets ___, and Trixie gets ___.

Problems 34-41 are multiple choice.

Problems 34-39 refer to the Selfridge-Conway envy free procedure for 3 players whose steps are given below.
Stage 1: The initial division
Step 1: Player 1 cuts the cake into, what in his view, is 3 equal pieces.
Step 2: Player 2, if he thinks one piece is largest, trims from that piece to create what he believes is a 2-way tie for largest piece. The trimmings are set aside. If player 2 thinks that the original split was fair, he does nothing.
Step 3: Player 3 may choose any piece.
Step 4: Player 2 chooses a piece. If the trimmed piece remains, he must choose it. If not, he chooses the one he feels is tied with the trimmed piece for largest.
Step 5: Player 1 gets the remaining piece.

Stage 2: Dividing the trimmings. Assume player 3 received the trimmed piece in stage 1.
Step 6: Player 2 divides the trimmings into what he considers 3 equal parts.
Step 7: Player 3 chooses one part of the trimmings.
Step 8: Player 1 chooses a piece of the trimmings.
Step 9: Player 2 receives the remaining trimmings.
34. Which one of the following explains why player 1 is envy-free after **stage 1**?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and
the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he is not
envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is
available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player
3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the
trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.

35. Which one of the following explains why player 2 is envy-free after **stage 1**?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and
the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he is not
envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is
available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player
3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the
trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.

36. Which one of the following explains why player 3 is envy-free after **stage 1**?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and
the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he is not
envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is
available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player
3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the
trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.

37. Which one of the following explains why player 1 is envy-free after **stage 2**?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and
the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he isn’t envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is
available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player
3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the
trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.
38. Which one of the following explains why player 2 is envy-free after stage 2?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he isn’t envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player 3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.

39. Which one of the following explains why player 3 is envy-free after stage 2?
A) He felt all 3 pieces were equal until the trimming was done, so he now feels two are equal and the trimmed piece is smaller. Since the trimmed piece must be gone after step 4, he is not envious.
B) He is not envious since he created a 2-way tie for first and at least one of those two pieces is available when it is his turn to pick.
C) He is not envious since he had first choice.
D) He does not envy player 2 because he is choosing ahead of player 2. He does not envy player 3 because player 3 has the trimmed piece and player 1 considers the trimmed piece plus all of the trimmings to be only one-third of the whole.
E) He envies no one because he made all 3 pieces of the trimmings equal in step 6.

40. Two people use the divide-and-choose procedure to divide a field. Suppose Jose divides and Maria chooses. Which statement is true?
A) Maria can guarantee that she always gets at least her fair share.
B) Maria always believes she gets more than her fair share.
C) Maria can possibly believe she gets less than her fair share.
D) There is an advantage to being the divider.
E) None of the above.

41. Suppose we have three items X, Y, and Z and three people Moe, Larry, and Curly. Assume that each of the people spreads 100 points over the items (as in the adjusted winner procedure) to indicate the relative worth of each item to that person.

<table>
<thead>
<tr>
<th>Item</th>
<th>Moe</th>
<th>Larry</th>
<th>Curly</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>40</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Y</td>
<td>50</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Z</td>
<td>10</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

Suppose Moe gets Z, Larry gets Y, and Curly gets X. This allocation is not Pareto-optimal. Find another allocation that makes one person better off without making anyone else worse off.
A) Moe gets X, Larry gets Y, and Curly gets Z
B) Moe gets Z, Larry gets X, and Curly gets Y
C) Moe gets X, Larry gets Z, and Curly gets Y
D) Moe gets X and Y, Larry gets Z, and Curly gets nothing
E) It is impossible to find another allocation that makes one person better off without making anyone else worse off.
CHAPTER 14

42. A country is divided into three states with the following populations:

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>2,390</td>
</tr>
<tr>
<td>Central</td>
<td>1,885</td>
</tr>
<tr>
<td>South</td>
<td>852</td>
</tr>
</tbody>
</table>

There are 26 seats in the national assembly. What is North's lower quota?

43. A country with 5 states has the following population figures.

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>9061</td>
</tr>
<tr>
<td>South</td>
<td>7179</td>
</tr>
<tr>
<td>East</td>
<td>5259</td>
</tr>
<tr>
<td>West</td>
<td>3319</td>
</tr>
<tr>
<td>Central</td>
<td>1182</td>
</tr>
<tr>
<td>Total</td>
<td>26,000</td>
</tr>
</tbody>
</table>

How should the 26 seats be apportioned among the 5 states if the apportionment method used is:

a) Hamilton’s?   
b) Jefferson’s?   
c) Webster’s?   
d) Hill-Huntington?

44. The 1970 census showed Florida had a population of 6,855,702 and Georgia had a population of 4,627,306. Florida was apportioned 15 seats and Georgia was apportioned 10 seats. Give the following answers to one decimal place.

a) Find Florida’s district population.  
b) Find Georgia’s district population.  
c) Which state is more favored in this apportionment?  
d) What is the relative difference in the district populations?  

45. Which of the apportionment method(s) we studied:

a) is currently used to apportion the U.S. House of Representatives?  
b) never violate the quota condition?  
c) avoid the population paradox and satisfy the quota condition?  
d) are susceptible to occurrences of the Alabama paradox?

46. The 1790 census showed Delaware had a population of 55,540 and Virginia had a population of 630,560. Delaware was apportioned 1 seat and Virginia was apportioned 19 seats.

a) Find Delaware’s representative share (in microseats/person, rounded to one decimal place).  
b) Find Virginia’s representative share (in microseats/person, rounded to one decimal place).  
c) Which state is more favored in this apportionment?  
d) What is the relative difference in the representative shares, rounded to one decimal place?

47. A state has a quota of 13.561. If the Hill-Huntington method is used, what is the cutoff for rounding? In other words, we round down if the quota is below what number? Round your answer to 3 decimal places.

48. A state has a quota of 7.499. Round this quota using:

a) Webster's method   
b) Jefferson’s method   
c) Hill-Huntington method
49. Find the relative difference of 189 and 509, rounded to 2 decimal places.

50. Find the geometric mean of 27 and 28, rounded to 3 decimal places.

Problems 51-56 are multiple choice.

51. A country is divided into three states with the following populations:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>1,264</td>
</tr>
<tr>
<td>Central</td>
<td>932</td>
</tr>
<tr>
<td>South</td>
<td>164</td>
</tr>
</tbody>
</table>

There are 10 seats in the national assembly. What is the standard divisor?
A) 5.36 B) 3.95 C) 0.69 D) 786.67 E) 236

52. A country is divided into three states with the following populations:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>1,890</td>
</tr>
<tr>
<td>Central</td>
<td>2,154</td>
</tr>
<tr>
<td>South</td>
<td>758</td>
</tr>
</tbody>
</table>

There are 25 seats in the national assembly. What is Central's upper quota?
A) 12 B) 11 C) 11.21 D) 192.08 E) 2154

53. A state has a population of 414,742 and was apportioned 9 seats in the House of Representatives. What is this state’s representative share? Express your answer in microseats/person.
A) 46082.44 B) 21.7 C) 2.17 D) 0.05 E) 0.0000217

54. Find the absolute difference of 189 and 509.
A) 320 B) -169.31% C) -320 D) 169.31% E) 1.69%

55. A country is divided into four states with the following populations:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>500</td>
</tr>
<tr>
<td>East</td>
<td>460</td>
</tr>
<tr>
<td>West</td>
<td>410</td>
</tr>
<tr>
<td>South</td>
<td>330</td>
</tr>
</tbody>
</table>

There are 10 seats in the national assembly. If Hamilton’s method is used, which state(s) get an extra seat?
A) North B) South C) North & East D) North & South E) North, East & South
56. Consider a small country with three states and the following census data:

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10,030</td>
</tr>
<tr>
<td>B</td>
<td>9030</td>
</tr>
<tr>
<td>C</td>
<td>940</td>
</tr>
</tbody>
</table>

When Hamilton’s method is used to apportion 200 seats, the result is
A gets 100 seats, B gets 90 seats, and C gets 10 seats.
When Hamilton’s method is used to apportion 201 seats, the result is
A gets 101 seats, B gets 91 seats, and C gets 9 seats.
Which ONE of the following explains why these apportionments show Hamilton’s method is susceptible to the Alabama paradox?
A) In the second apportionment, C got fewer seats than its lower quota.
B) If state A and state B each gain a seat, so should state C.
C) State C lost a seat to state B even though the population of C had grown at a faster rate than that of B.
D) Although the House size increased, state C lost a seat.
E) There is not enough information since we don’t know which state Alabama is.

CHAPTER 15

57. In the game of batter vs. pitcher, the pitcher has two strategies: throw a fastball or throw a curve. The batter also has two strategies: guess a fastball or guess a curve. The entries in the matrix are the batter's batting averages, which are the probabilities the batter gets a hit.

<table>
<thead>
<tr>
<th></th>
<th>Pitcher throws fastball</th>
<th>Pitcher throws curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batter guesses fastball</td>
<td>0.500</td>
<td>0.200</td>
</tr>
<tr>
<td>Batter guesses curve</td>
<td>0.200</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Find the proportion of the time the pitcher should throw a fastball.

58. In the following game of batter-versus-pitcher in baseball, the batter’s batting averages are shown in the payoff matrix:

<table>
<thead>
<tr>
<th></th>
<th>Pitcher throws fastball</th>
<th>Pitcher throws curve</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batter guesses fastball</td>
<td>0.300</td>
<td>0.100</td>
</tr>
<tr>
<td>Batter guesses curve</td>
<td>0.200</td>
<td>0.400</td>
</tr>
</tbody>
</table>

The game has already been solved for you and the optimal strategies are:
The batter should guess fastball ½ the time and guess curve ½ the time.
The pitcher should throw a fastball ¾ of the time and throw a curve ¼ of the time.

Find the value of the game.

59. Consider the following zero sum game in which the ROW’s optimal strategy is to play strategy \( R_1 = \frac{1}{2} \) of the time and strategy \( R_2 = \frac{1}{2} \) of the time. COLUMN’s best strategy is to play strategy \( C_1 = \frac{1}{4} \) of the time and strategy \( C_2 = \frac{3}{4} \) of the time. What is the value of this game?

\[
R_1 = \begin{bmatrix} .4 & .2 \\ .1 & .3 \end{bmatrix} 
\]

\[
C_1 = \begin{bmatrix} .4 & .2 \\ .1 & .3 \end{bmatrix} 
\]
60. Consider the following 2-person, zero-sum game:

\[
\begin{bmatrix}
R_1 & C_1 & C_2 \\
4 & 2 & \\
9 & 6 & \\
\end{bmatrix}
\]

What is each player’s best strategy?

61. Consider the following 2-person, zero-sum game:

\[
\begin{bmatrix}
R_1 & C_1 & C_2 \\
3 & 5 & \\
4 & 2 & \\
\end{bmatrix}
\]

What is the best strategy for the ROW player? You do not have to find the COLUMN player’s best strategy.

62. Consider the following 2-person, zero-sum game. Use the idea of dominant strategies to cross out any rows or columns that represent strategies that should never be played. Do not bother checking for a saddlepoint and do not solve the smaller game that results.

\[
\begin{bmatrix}
1 & 5 & 7 & 3 \\
6 & 2 & 14 & 12 \\
15 & 9 & 16 & 4 \\
10 & 11 & 18 & 13 \\
\end{bmatrix}
\]

63. Consider the following 2-person, zero-sum game. Use the idea of dominant strategies to cross out any rows or columns that represent strategies that should never be played. Do not bother checking for a saddlepoint and do not solve the smaller game that results.

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>R2</td>
<td>-1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>R3</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

64. You have the choice of either parking illegally on the street or parking in the lot and paying $10. Parking illegally is free if the police officer is not patrolling, but you receive a $40 parking ticket if she is. However, you are peeved when you pay to park in the lot on days when the officer does not patrol, and you assess this outcome as costing $20 ($10 for parking plus $10 for your time, inconvenience, and grief). Write a matrix assuming this is a zero-sum game with you as the row player. You do not have to solve the game.

65. An election has 3 voters X, Y, and Z and three alternatives x, y and z. The method of voting used is plurality and the chair X has the tie-breaking vote. The preference schedules for each voter are X prefers x to z to y, indicated by xzy, Y’s preference is yxz and Z’s preference is zxy.

a) What is X’s dominant strategy?
b) Given that X votes its dominant strategy, write the reduced 3 x 3 payoff matrix for Y vs. Z.
c) Eliminate all dominated rows and columns to get a second reduced payoff matrix.
d) What is the outcome of this election under sophisticated voting?
e) Is the following outcome a Nash equilibrium? Justify your answer.

X votes for x, Y votes for z, and Z votes for z.
66. Two firms, which we will call ROW and COLUMN, are seeking the same government contract. Each firm has two strategies: to hire lobbyists or to not hire lobbyists. Lobbying entails a cost of 15. Not lobbying costs nothing. If both firms lobby or neither firm lobbies, then the government makes a neutral decision, which yields 10 to both firms. A firm’s payoff is this value minus the lobbying cost, if it lobbied. If COLUMN lobbies and ROW does not lobby, then the government makes a decision that favors COLUMN, yielding zero to ROW and 30 to COLUMN. Thus, COLUMN’s payoff in this case is 30-15 = 15. If ROW lobbies and COLUMN does not lobby, then the government makes a decision that favors ROW, yielding 40 to ROW and zero to COLUMN. The payoff matrix for this game is shown below.

<table>
<thead>
<tr>
<th>Lobby</th>
<th>Don’t lobby</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lobby</td>
<td>(-5, -5)</td>
</tr>
<tr>
<td>Don’t lobby</td>
<td>(0, 15)</td>
</tr>
</tbody>
</table>

a) Does either player have a dominant strategy?
b) Find all Nash equilibria.

67. During the period 1980-1 in Poland, the Solidarity labor union challenged the ruling Communist party. The party had two choices: reject (R) or accept (A) the limited autonomy of plural social forces set loose by Solidarity. Rejection would, if successful, restore the monolithic structure underlying Communist rule. Acceptance would allow political institutions other than the Communist party to participate in some meaningful way in the formulation of public policy. Solidarity also had two strategies: reject (R) or accept (A) the monolithic structure of the country. Rejection would put pressure on the government to limit severely the extent of the state’s authority in political matters. Acceptance would significantly reduce the chances of Solidarity or other independent institutions to alter certain state activities. The two strategies available to each side give rise to four possible outcomes, with the associated payoffs being the rankings of each possible outcome as theorized by NYU political scientist Steven J. Brams:

<table>
<thead>
<tr>
<th></th>
<th>Communist party R</th>
<th>Communist party A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solidarity A</td>
<td>(2, 4)</td>
<td>(3, 3)</td>
</tr>
<tr>
<td>Solidarity R</td>
<td>(1, 2)</td>
<td>(4, 1)</td>
</tr>
</tbody>
</table>

a) Does either player have a dominant strategy?
b) Find all Nash equilibria.

68. Find each player’s optimal strategy in the following game:

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>R2</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

A) ROW should always use R1 and COLUMN should always use C1
B) ROW should always use R2 and COLUMN should always use C1
C) ROW should always use R1 and COLUMN should always use C2
D) ROW should always use R2 and COLUMN should always use C2
E) The optimal strategy for each player is a mixed strategy, not a pure strategy
69. In the following zero-sum game, the payoffs represent gains to the ROW player.
\[
\begin{pmatrix}
3 & 5 \\
4 & 2
\end{pmatrix}
\]
Which of the following statements is true?
A) The game has no saddle point.
B) The game has a saddle point and the value of the game is 2.
C) The game has a saddle point and the value of the game is 3.
D) The game has a saddle point and the value of the game is 4.
E) The game has a saddle point and the value of the game is 5.

70. Consider the following two-person game between players ROW and COLUMN. ROW has two possible strategies R₁ and R₂. COLUMN has the strategies C₁ and C₂.
\[
\begin{array}{cc}
C₁ & C₂ \\
R₁ & -5 & 8 \\
R₂ & -21 & 12
\end{array}
\]
Which one of the following is true?
A) The saddlepoint is -5
B) The saddlepoint is 8
C) The saddlepoint is -21
D) The saddlepoint is 12
E) The game has no saddlepoint

71. A fair game is a game:
A) that has a saddlepoint.
B) that has a value of 0.
C) in which both players have an optimal pure strategy.
D) in which both players have an optimal mixed strategy.
E) None of the above.

72. Consider the following two-person game in which the entries in the matrix represent payoffs to the ROW player.
\[
\begin{array}{ccc}
 & C₁ & C₂ & C₃ \\
R₁ & 1 & 4 & -1 \\
R₂ & 1 & 5 & 2 \\
R₃ & 1 & 3 & 0
\end{array}
\]
Which row(s) can be crossed out because they represent dominated strategies?
A) R₁  B) R₂  C) R₃  D) R₁ & R₂  E) R₁ & R₃

73. Consider the following two-person non-zero sum game between ROW and COLUMN.
\[
\begin{array}{cc|cc}
 & C₁ & C₂ \\
ROW uses R₁ & (1, 2) & (4, 2) \\
ROW uses R₂ & (3, 4) & (2, 0)
\end{array}
\]
Find all Nash equilibria:
A) (4, 2)  B) (3, 4)  C) (2, 0)
D) both (4, 2) and (3, 4)
E) There is no Nash equilibrium in this game.
74. An election has 3 voters Ingrid, Javier, and Katie, and three alternatives a, b and c. The method of voting used is plurality and the chair Ingrid has the tie-breaking vote. The preference schedules for each voter are Ingrid prefers a to b to c, indicated by abc, Javier’s preference is bac, and Katie's preference is cba. Is the following outcome a Nash equilibrium?

Ingrid votes for a, Javier votes for b, and Katie votes for c.

A) No. Alternative a wins the 1-1-1 vote because Ingrid is the tiebreaker. But, if Katie changes her vote to b, alternative b would win, which is an improvement over her last preference.
B) Yes. No one can improve by unilaterally changing their vote because regardless of how they vote, they will still be outvoted 2-1, leaving a the winner.
C) Yes. Alternative a wins. Ingrid cannot improve her outcome because she is outvoted by Katie and Javier. Javier and Katie cannot improve their outcomes because they already have their #1 preference.
D) No. Alternative a wins the 1-1-1 vote because Ingrid is the tiebreaker. But, if Katie changes her vote to a or b, alternative a would win. This would be an improvement from her third preference to her second.
E) Yes. Alternative a wins the vote 1-1-1 because Ingrid is the tiebreaker. Ingrid and Javier cannot improve their outcome because their #1 preference already won. Katie cannot improve her outcome because she is outvoted by Ingrid and Javier.

ANSWERS

1a) Amy   b) Emilia   c) Carla   d) Doris   e) Betsy
2a) 61   b) 71
3) independence of irrelevant alternatives
4) Condorcet winner criterion
5) sequential pairwise voting
6a) B   b) B, D and F
7) monotonicity
8) Pareto condition
9a) C first, then A, so B wins   b) 18   c) B
10) C
11) D
12) E
13) D
14) A
15) D
16a) dummy   b) dictator   c) veto power
17) 2
18) A
19) {A, D}, {A, C} and {B, C, D}
20a) \((\frac{2}{3}, \frac{1}{6}, \frac{1}{6})\)   b) \((\frac{1}{2}, \frac{1}{2}, 0)\)   c) \((\frac{2}{7}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42}, \frac{5}{42})\)
21a) (6, 2, 2)   b) (4, 4, 0)
22) C
23) B
24) D
25) A
26) E
27) A

28) Ivana gets the Conn. Estate, the TP apartment, the cash & jewelry, and \( \frac{2}{15} \) of the PB mansion. Donald gets the TT Triplex and \( \frac{13}{15} \) of the PB mansion.

29) Alex gets the textbooks, barbells, desk and \( \frac{16}{21} \) of the photos. Jose gets the bike, rowing machine, music, computer, and \( \frac{5}{21} \) of the photos.
30) John gets the boat and $39,375 cash, Paul gets $74,375 cash, George gets the house and pays $168,125 and Ringo gets the car and $54,375 cash.
31) A gets the jewelry and $56,000 cash, B gets the car and $34,000 cash, and C gets the house and pays $90,000.
32a) Yes, to make Javier better off, Mary would end up worse off.
32b) No, Javier does not get at least 50% of the whole.
32c) No, Javier envies Mary.
32d) No, Mary perceives her share to be greater than Javier perceives his share.
32e) No. By giving Javier Y and Mary X, we make Javier better off without making Mary worse off.
33a) Yes, all gets at least \( \frac{1}{4} \) of the whole.
33b) Alice envies Ed because he received W, which is what she valued most.
33c) No, the only way to make Alice better off is to give her W, which makes Ed worse off.
33d) Ralph gets X, Alice gets Z, Ed gets Y, and Trixie gets W.

34) A
35) B
36) C
37) D
38) E
39) C
40) A
41) A
42) 12

43a) North 9, South 7, East 5, West 4, Central 1
43b) North 10, South 7, East 5, West 3, Central 1
43c) North 9, South 8, East 5, West 3, Central 1
43d) North 9, South 7, East 6, West 3, Central 1

44a) 457,046.8 b) 462,730.6 c) Florida d) 1.2%
45a) Hill-Huntington b) Hamilton c) none d) Hamilton
46a) 18.0 b) 30.1 c) Virginia d) 67.2%

47) 13.491

48a) 7 b) 7 c) 8

49) 169.31%
50) 27.495
51) E
52) A
53) B
54) A
55) E
56) D
57) ¼
58) 0.25 or ¼
59) 0.25 or ¼
60) ROW should always use strategy R₂ and COLUMN should always use strategy C₂.
61) ROW should use strategy R₁ ½ of the time and use strategy R₂ ½ of the time.
62) Cross our R₁ and R₂ and cross out C₃.
63) Cross our R₁ and R₂ and cross out C₃.
64) Police

\[
\begin{bmatrix}
\text{street} & \text{patrols} & \text{no patrol} \\
\text{lot} & -40 & 0 \\
& -10 & -20
\end{bmatrix}
\]

65a) vote for x
65b) Z votes for

\[
\begin{bmatrix}
x & y & z \\
x & x & x \\
z & x & x \\
\end{bmatrix}
\]

Y votes for

\[
\begin{bmatrix}
x & y & x \\
z & x & x \\
\end{bmatrix}
\]
65c) Z votes for

\[
\begin{bmatrix}
z \\
z \\
\end{bmatrix}
\]

Y votes for y[x]
65d) x wins
65e) No, since z wins 2-1 and if Y unilaterally changes his vote to x or y, x will win, which Y considers an improvement over z winning.
66a) No
66b) (0, 15) and (25, 0)
67a) The Communist party has a dominant strategy of rejection.
67b) The only Nash equilibrium is the outcome (2, 4)
68) C
69) A
70) A
71) B
72) E
73) D
74) A