Exact value of $\sin 18^\circ = \cos 72^\circ$

$\sin 36^\circ = \cos 54^\circ$. Note $36^\circ = 2(18^\circ)$, $54^\circ = 3(18^\circ)$

Recall $\sin 2x = 2\sin x \cos x$, and

$\cos 3x = 4\cos^3 x - 3\cos x$. Therefore

$\sin 36^\circ = \sin 2(18^\circ) = 2\sin 18^\circ \cos 18^\circ$, $\cos 54^\circ = \cos 3(18^\circ) = 4\cos^3 18^\circ - 3\cos 18^\circ$

So $2\sin 18^\circ \cos 18^\circ = 4\cos^3 18^\circ - 3\cos 18^\circ$

Divide by $\cos 18^\circ$

$$2\sin 18^\circ = 4\cos^2 18^\circ - 3$$

$$2\sin 18^\circ = 4(1 - \sin^2 18^\circ) - 3$$

$$2\sin 18^\circ = 4 - 4\sin^2 18^\circ - 3$$

$$4\sin^2 18^\circ + 2\sin 18^\circ - 1 = 0$$

By the Quadratic Formula,

$$\sin 18^\circ = \frac{-2 \pm \sqrt{4 - 4(4)(-1)}}{2(4)}$$

$$= \frac{-2 \pm \sqrt{20}}{8}$$

$$= \frac{-2 \pm 2\sqrt{5}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

Reject $\frac{-1-\sqrt{5}}{4}$ since $\sin 18^\circ > 0$

So $\sin 18^\circ = \frac{-1+\sqrt{5}}{4}$

and also $\cos 72^\circ = \frac{-1+\sqrt{5}}{4}$