MAC 1114 EXAM VI MR. NADEL FALL 2009

(5) ① Find $(-\sqrt{2} + i\sqrt{2})^4$ directly using the Binomial Theorem.

(15) ② a) Find $(-\sqrt{2} + i\sqrt{2})^4$ using DeMoivre's Thm. Put answer in exact rectangular form.

b) Find all cube roots of $64i$ exactly. Put answers in $a + bi$ form.

c) Find all fourth roots of -16 in polar form.

(10) ③ a) Sketch and find the focus of $y^2 = -8x$.

b) Find the equation of a parabola with vertex at the origin, its axis the y axis, and $(-2, 3)$ on its graph.

(10) ④ a) Sketch
$$16x^2 + 36y^2 = 576$$
b) Find the coordinates of its foci.

(15) ⑤ a) Sketch
$$\frac{y^2}{4} - \frac{x^2}{25} = 1$$
b) Find the coordinates of its foci.

c) Write the equations of its asymptotes.

(15) ⑥ a) Sketch
5 pts. $y - 2 = \frac{1}{2}(x+3)^2$
Use only integer coordinates.

b) Find the center and identify the type of conic:
10 pts.
$$16x^2 + 9y^2 + 96x - 18y = -9.$$
Don't sketch.

MAC 1114 EXAM VI KEY (F '09)

① ~~a~~) $(-\sqrt{2})^4 + 4(-\sqrt{2})^3(i\sqrt{2}) + 6(-\sqrt{2})^2(i\sqrt{2})^2$
$+ 4(-\sqrt{2})(i\sqrt{2})^3 + (i\sqrt{2})^4$
$= 4 - 16i - 24 + 16i + 4 = -16$

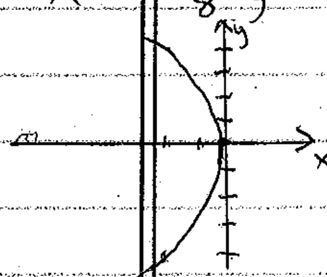
② a) $(2 \text{ cis } 135^\circ)^4$
$= 16 \text{ cis } 540^\circ$
$= 16 \text{ cis } 180^\circ = -16$

b) $64i = 64 \text{ cis } 90^\circ$
$64^{1/3} \text{ cis}\left(\frac{90^\circ + 360^\circ k}{3}\right)$
$k = 0, 1, 2$
$4 \text{ cis } 30^\circ = 2\sqrt{3} + 2i$
$4 \text{ cis } 150^\circ = -2\sqrt{3} + 2i$
$4 \text{ cis } 270^\circ = -4i$

c) $-16 = 16 \text{ cis } 180^\circ$
$16^{1/4} \text{ cis}\left(\frac{180^\circ + 360^\circ k}{4}\right)$
$2 \text{ cis } 45^\circ, 2 \text{ cis } 135^\circ,$
$2 \text{ cis } 225^\circ, 2 \text{ cis } 315^\circ$

③ a) $4a = -8 \Rightarrow a = -2$
Focus is $(-2, 0)$
$x = -\frac{1}{8}y^2$

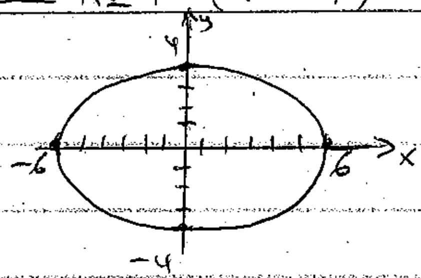
y	x
0	0
±4	−2



parabola

b) $x^2 = 4ay$
$(-2)^2 = 4a(3)$
$\frac{4}{3} = 4a$
$x^2 = \frac{4}{3}y$

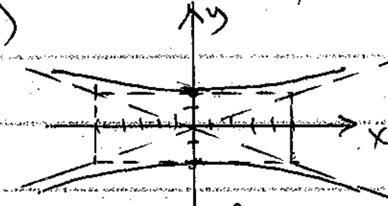
④ a)



ellipse

b) $c^2 = a^2 - b^2 = 36 - 16 = 20$
$c = \sqrt{20}$
foci $(\pm\sqrt{20}, 0)$ or $(\pm 2\sqrt{5}, 0)$

⑤ a)



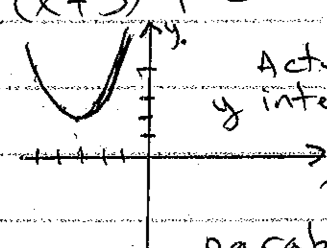
hyperbola

b) $c^2 = a^2 + b^2 = 4 + 25 = 29$
foci $(0, \pm\sqrt{29})$

c) $y = \pm\frac{2}{5}x$ from the graph

⑥ a) $y = \frac{1}{2}(x+3)^2 + 2$

x	y
−7	10
−5	4
−3	2
−1	4
1	10



Actually, y intercept is $6\frac{1}{2}$.

parabola

b) $16x^2 + 96x + 9y^2 - 18y = -9$
$16(x^2 + 6x + 9) + 9(y^2 - 2y + 1)$
$= -9 + 144 + 9 = 144$
$$\frac{(x+3)}{9} + \frac{(y-1)^2}{16} = 1$$

ellipse centered at $(-3, 1)$