MAC 1114 EXAM II MR. NADEL FALL 2011

(10) ① Prove

$$\frac{\sin\theta + \tan\theta}{1 + \sec\theta} = \sin\theta$$

(15) ② a) Simplify $\cos\left(x + \frac{\pi}{3}\right)$ completely.

b) Find the exact value of $\sin 105^\circ$ using an addition formula.

c) Find the exact value of

$$\frac{\tan 40^\circ - \tan 10^\circ}{1 + \tan 40^\circ \tan 10^\circ}$$

(10) ③ Derive a formula for $\tan(x-y)$ in terms of $\tan x$ and $\tan y$. (Prove it)

(6) ④ a) If $\sin x = -\frac{3}{7}$, $\pi \le x \le \frac{3\pi}{2}$ find the exact value of $\sin 2x$

b) Find the exact value of $\cos 67.5^\circ$ using a half-angle formula. Simplify.

(10) ⑤ Prove

$$\tan x \sin 2x = 2 - 2\cos^2 x$$

(10) ⑥ Derive

$$\cos 3x = 4\cos^3 x - 3\cos x$$

(10) ⑦ a) Find the exact value of $\cos 195^\circ \cdot \cos 105^\circ$ using a product-sum identity.

b) Write as a product: $\sin 7x - \sin 3x$. Simplify.

(25) ⑧ a) Solve $\sin\theta = -.315$ on $0^\circ \le \theta \le 360^\circ$. (5 pts.)

b) Solve $2\sin 2x = \sqrt{2}$ on $0 \le x \le \pi$. (4 pts.)

c) Solve $2\cos^2 x - \sin x = 1$ on $0^\circ \le x \le 360^\circ$ (8 pts.)

d) Solve $\sin 2x = \sin x$ on $-\pi \le x \le \pi$. (8 pts.)

MAC 1114 EXAM II KEY (F '11)

① $\dfrac{\sin\theta+\tan\theta}{1+\sec\theta} = \dfrac{\left(\sin\theta+\frac{\sin\theta}{\cos\theta}\right)}{\left(1+\frac{1}{\cos\theta}\right)}\dfrac{\cos\theta}{\cos\theta}$

$= \dfrac{\sin\theta\cos\theta+\sin\theta}{\cos\theta+1}$

$= \dfrac{\sin\theta(\cos\theta+1)}{\cos\theta+1} = \sin\theta$

②a) $\cos x\cos\frac{\pi}{3} - \sin x\sin\frac{\pi}{3}$

$= \cos x\left(\frac{1}{2}\right) - \sin x\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

b) $\sin(60°+45°)$

$= \sin 60°\cos 45° + \cos 60°\sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{6}+\sqrt{2}}{4}$

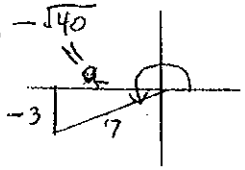
c) By formula,

$\tan(40°-10°) = \tan 30°$

$= \frac{1}{\sqrt{3}}$ OR $\frac{\sqrt{3}}{3}$

③ see notes, text

④a) $-\sqrt{40}$



$a^2+9=49 \Rightarrow a=-\sqrt{40}$

$\sin 2x = 2\sin x\cos x$

$= 2\left(-\frac{3}{7}\right)\left(-\frac{\sqrt{40}}{7}\right)$

$= \frac{6\sqrt{40}}{49}$ OR $\frac{12\sqrt{40}}{49}$

b) $\frac{1}{2}x = 67.5°$ $\quad x = 135°$

$\cos 67.5° = \sqrt{\dfrac{1+\cos 135°}{2}}$

$= \sqrt{\dfrac{\left(1-\frac{\sqrt{2}}{2}\right)\cdot\frac{2}{2}}{2}}$

$= \dfrac{\sqrt{2-\sqrt{2}}}{2}$

⑤ $\tan x\sin 2x = \tan x(2\sin x\cos x)$

$= \dfrac{\sin x}{\cos x}(2\sin x\cos x)$

$= 2\sin^2 x$

$= 2(1-\cos^2 x)$

$= 2-2\cos^2 x$

⑥ see handout online

⑦a) $\frac{1}{2}[\cos(195°+105°) + \cos(195°-105°)]$

$= \frac{1}{2}[\cos 300° + \cos 90°]$

$= \frac{1}{2}\left[\frac{1}{2}+0\right] = \frac{1}{4}$

b) $2\cos\left(\dfrac{7x+3x}{2}\right)\sin\left(\dfrac{7x-3x}{2}\right)$

$= 2\cos 5x\sin 2x$

⑧a) ref. angle $= \sin^{-1}(+.315)$

$\alpha = 18.4°$ (degree mode)

$\theta = 198.4°$ (QIII) $(\alpha+180°)$

$\theta = 341.6°$ (QIV) $(360°-\alpha)$

b) $\sin 2x = \frac{\sqrt{2}}{2}$ $\quad 0 \le 2x \le 2\pi$

$2x = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{8}$

$2x = \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{8}$

c) $2\cos^2 x - \sin x = 1$

$2(1-\sin^2 x) - \sin x = 1$

$2-2\sin^2 x - \sin x = 1$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

$x = 30°, 150°$ $\quad x = 270°$

d) $2\sin x\cos x = \sin x$

$2\sin x\cos x - \sin x = \sin x(2\cos x - 1) = 0$

$\sin x = 0 \Rightarrow x = \pm\pi, 0$ $\quad \cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3}$.