(MWF) 65 pts.

MAC 1114 EXAM III MR. NADEL FALL 2011

(10) ① Suppose $\sin x = -\frac{2}{5}$ and $\pi \leq x \leq \frac{3\pi}{2}$. Find

a) $\sin 2x$ exactly

b) $\sin\left(\frac{1}{2}x\right)$ exactly.

(10) ② Prove

$$\cos 2x = \frac{\cot x - \tan x}{\cot x + \tan x}$$

(10) ③ Derive

$$\cos 3x = 4\cos^3 x - 3\cos x$$

(10) ④ a) Find the exact value of $\sin 157.5^\circ \cos 112.5^\circ$

b) Write as a product and simplify:

$\cos 3t + \cos t$

(25) ⑤ a) Solve $\tan\theta = -3.16$ (4 pts.) on $0^\circ \leq \theta \leq 360^\circ$.

b) Solve exactly (5 pts.)

$\sec^2 x - 4 = 0$

on $0 \leq x \leq 2\pi$.

c) Solve exactly (8 pts.)

$\cos 2x = \sin x$

on $0 \leq x \leq 2\pi$

d) Solve for θ on (8 pts.)

$0^\circ \leq \theta \leq 360^\circ$:

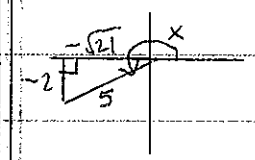
$\sin^2\theta - 2\sin\theta - 1 = 0$

Hint: It does not factor.

(MWF)

MAC 1114 (F'11) EXAM III KEY

① a) $\sin 2x = 2\sin x\cos x = 2\left(-\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right) = \frac{4\sqrt{21}}{25}$



b) $\pi \le x \le \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} \le \frac{1}{2}x \le \frac{3\pi}{4}$ (part of QII)

$$\sin\left(\frac{1}{2}x\right) = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{21}}{5}\right)}{2}}$$

(+ because in QII)

$$= \sqrt{\frac{5+\sqrt{21}}{10}}$$

② $$\frac{\cot x - \tan x}{\cot x + \tan x} = \frac{\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)}{\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)} \cdot \frac{\sin x\cos x}{\sin x\cos x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$$

$$= \frac{\cos 2x}{1} = \cos 2x$$

③ See printout online.

④ a) $\frac{1}{2}\left[\sin 270^\circ + \sin 45^\circ\right] = \frac{1}{2}\left[-1 + \frac{\sqrt{2}}{2}\right] = \frac{-2+\sqrt{2}}{4}$

b) $2\cos\left(\frac{3t+t}{2}\right)\cos\left(\frac{3t-t}{2}\right) = 2\cos 2t\cos t$

⑤ a) ref. angle $= 72.4^\circ$ $\quad \theta = 107.56^\circ$ (Q II)

$\theta = 287.56^\circ$ (QIV)

b) $\sec^2 x = 4 \Rightarrow \sec x = \pm 2$

$\cos x = \pm\frac{1}{2} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

c) $\cos^2 x - \sin^2 x = \sin x$

$1 - \sin^2 x - \sin^2 x = \sin x \Rightarrow 1 - 2\sin^2 x = \sin x$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$\sin x = \frac{1}{2}$ $\quad \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad x = \frac{3\pi}{2}$

d) $\sin\theta = \frac{2 \pm \sqrt{4-4(1)(-1)}}{2(1)} = \frac{2\pm\sqrt{8}}{2}$ Reject $\frac{2+\sqrt{8}}{2}$

$\sin\theta = \frac{2-\sqrt{8}}{2} \Rightarrow \alpha = \text{ref. angle} \approx 24.5^\circ$

$\theta = 204.47^\circ$ (Q III)

$\theta = 335.53^\circ$ (Q IV)