

Derivation of  $\cos 3x = 4\cos^3 x - 3\cos x$

$$\begin{aligned}\cos 3x &= \cos(2x+x) \\ &= \cos 2x \cos x - \sin 2x \sin x \\ &= (\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x \\ &= \cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x \\ &= \cos^3 x - 3 \sin^2 x \cos x \\ &= \cos^3 x - 3(1 - \cos^2 x) \cos x = \cos^3 x - 3(\cos x - \cos^3 x) \\ &= \cos^3 x - 3 \cos x + 3 \cos^3 x \\ &= 4 \cos^3 x - 3 \cos x\end{aligned}$$

Know this derivation and also

$$\sin 3x = 3 \sin x - 4 \sin^3 x$$