

(10) ① a) Find the exact value of $\cos 2x$ if $\cos x = \frac{1}{3}$ and x is in QII.

b) Find the exact value of $\cos 112.5^\circ$ using a half angle formula.
No compound fractions.

(25) ⑤ a) Solve to 2 dec. places:
4 pts. $\cos \theta = -.342$
on $0^\circ \leq \theta \leq 360^\circ$.

b) Solve exactly on
5 pts. $0 \leq x < 2\pi$.
 $\cos 2x = \frac{\sqrt{3}}{2}$.

(10) ② Prove
 $\frac{\sin^2 2\theta}{\sin^2 \theta} = 4 - 4\sin^2 \theta$

c) Solve
8 pts. $2\sin^2 \theta - \cos \theta - 1 = 0$
on $0 \leq \theta \leq 360^\circ$.

Hint: Write in terms of $\cos \theta$.

(10) ③ Derive
 $\sin 3x = 3\sin x - 4\sin^3 x$.

d) Solve
8 pts. $\sin \theta = 1 - \cos \theta$ on
 $0 \leq \theta < 2\pi$.

Hint: Square both sides.

Be sure to check answers.

It's not $\sin^2 \theta = 1 - \cos^2 \theta$.

Don't forget the middle term.

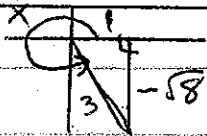
(10) ④ a) Write $\sin 4\theta \cos 3\theta$ as a sum.

b) Evaluate

$\cos 105^\circ \cos 75^\circ$
exactly.

MAC 1114 EXAM III KEY (F'09)

① a)



$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(\frac{1}{3}\right)^2 - \left(-\frac{\sqrt{8}}{3}\right)^2 \\ &= \frac{1}{9} - \frac{8}{9} = -\frac{7}{9} \end{aligned}$$

b)

$$= -\sqrt{\frac{1 + \cos 225^\circ}{2}} = -\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{-\sqrt{2 - \sqrt{2}}}{2}$$

$$\begin{aligned} \frac{\sin^2 2x}{\sin^2 x} &= \frac{(2 \sin x \cos x)^2}{\sin^2 x} \\ &= \frac{4 \cancel{\sin^2 x} \cos^2 x}{\cancel{\sin^2 x}} \\ &= 4 \cos^2 x \\ &= 4(1 - \sin^2 x) = 4 - 4 \sin^2 x \end{aligned}$$

③ see text, notes

$$\begin{aligned} \text{④ a)} \quad &\frac{1}{2} [\sin(4\theta + 3\theta) + \sin(4\theta - 3\theta)] \\ &= \frac{1}{2} (\sin 7\theta + \sin \theta) \end{aligned}$$

$$\begin{aligned} \text{b)} \quad &\frac{1}{2} [\cos 180^\circ + \cos 30^\circ] \\ &= \frac{1}{2} \left[-1 + \frac{\sqrt{3}}{2}\right] = \frac{\sqrt{3} - 2}{4} \end{aligned}$$

⑤ a) $x = 70^\circ = \text{ref angle}$

(use $\cos^{-1}(+.342)$)

$$\theta = 110^\circ \quad \text{QII}$$

$$\theta = 250^\circ \quad \text{QIII}$$

⑤ b) $\cos 2x = \frac{3}{2}$

$$2x = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12}$$

$$2x = \frac{11\pi}{6} \Rightarrow x = \frac{11\pi}{12}$$

$$2x = \frac{13\pi}{6} \Rightarrow x = \frac{13\pi}{12}$$

$$2x = \frac{23\pi}{6} \Rightarrow x = \frac{23\pi}{12}$$

⑤ c) $2 \sin^2 x - \cos x - 1 = 0$

$$2(1 - \cos^2 x) - \cos x - 1 = 0$$

$$2 - 2 \cos^2 x - \cos x - 1 = 0$$

$$0 = 2 \cos^2 x + \cos x - 1$$

$$0 = (2 \cos x - 1)(\cos x + 1)$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$x = 60^\circ, 300^\circ \quad x = 180^\circ$$

⑤ d) $\sin \theta = 1 - \cos \theta$

$$\sin^2 \theta = (1 - \cos \theta)^2$$

$$\sin^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$$

$$1 - \cos^2 \theta = 1 - 2 \cos \theta + \cos^2 \theta$$

$$0 = 2 \cos^2 \theta - 2 \cos \theta$$

$$2 \cos \theta (\cos \theta - 1) = 0$$

$$\cos \theta = 0 \quad \cos \theta = 1$$

$$\theta = \frac{\pi}{2}, \left(\frac{3\pi}{2}\right) \quad \theta = 0$$

↑
reject