MAC 1114 EXAM II MR. NADEL SPRING 2011

(10)① Prove

$$\frac{1}{1-\cos x}+\frac{1}{1+\cos x}=2\csc^2 x$$

(15)② a) Simplify completely:

$\tan\left(x+\frac{\pi}{3}\right)$

Use exact value(s).

b) Use an appropriate identity to find the exact value of

$\sin 22^\circ \cos 38^\circ + \cos 22^\circ \sin 38^\circ$

c) Find the exact value of $\cos 75^\circ$ using an addition formula.

(10)③ Derive a formula for $\tan(x-y)$ in terms of $\tan x$ and $\tan y$. (Prove it.)

(10)④ a) If $\cos x = -\frac{1}{3}$, $\pi \le x \le \frac{3\pi}{2}$ find $\cos 2x$ exactly

b) Find $\cos 105^\circ$ exactly using a half angle formula.

(10)⑤ Prove

$$\tan\theta = \frac{\sin 2\theta}{1+\cos 2\theta}$$

(10)⑥ Derive

$\cos 3x = 4\cos^3 x - 3\cos x$

(10)⑦ a) Write as a sum:

$\sin 4x \cos 3x$

b) Find the exact value of $\sin 195^\circ + \sin 105^\circ$ using a sum-product identity.

(25)⑧ a) Solve $\sin\theta = -.218$ 5 pts. on $0^\circ \le \theta \le 360^\circ$.

b) Solve exactly 4 pts. $\sin 3x = 1$ on $0 \le x \le 2\pi$

c) Solve 8 pts. $\tan^2\theta - \tan\theta - 1 = 0$ on $0^\circ \le \theta \le 360^\circ$. (2 dec. places)

Hint: Use quadratic formula.

d) Solve 8 pts. $2\sin^2\theta - \cos 2\theta = 0$ on $0^\circ \le \theta \le 360^\circ$

Note:

"exact" or "exactly" means to leave multiples of π, radical form, etc. (i.e. not on the calculator)

MAC 1114 EXAM II KEY (SP '11)

① $\frac{1}{1-\cos x}+\frac{1}{1+\cos x}=\frac{1+\cos x+1-\cos x}{(1-\cos x)(1+\cos x)}$

$=\frac{2}{1-\cos^2 x}=\frac{2}{\sin^2 x}=2\csc^2 x$

② a) $\frac{\tan x+\tan\frac{\pi}{3}}{1-\tan x\tan\frac{\pi}{3}}=\frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}$

b) $\sin(22^\circ+38^\circ)=\sin 60^\circ=\frac{\sqrt{3}}{2}$

c) $\cos(30^\circ+45^\circ)=$

$\cos 30^\circ\cos 45^\circ-\sin 30^\circ\sin 45^\circ$

$=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$

③ See text, notes.

④ a)

$\cos 2x=\cos^2 x-\sin^2 x$

$=\left(-\frac{1}{3}\right)^2-\left(-\frac{\sqrt{8}}{3}\right)^2$

$=\frac{1}{9}-\frac{8}{9}=-\frac{7}{9}$

b) $\cos 105^\circ=-\sqrt{\frac{1+\cos 210^\circ}{2}}$ (QII)

$=-\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=-\frac{\sqrt{2-\sqrt{3}}}{2}$

⑤ $\frac{\sin 2\theta}{1+\cos 2\theta}=\frac{2\sin\theta\cos\theta}{1+\cos^2\theta-\sin^2\theta}$

$=\frac{2\sin\theta\cos\theta}{1-\sin^2\theta+\cos^2\theta}$

$=\frac{2\sin\theta\cos\theta}{\cos^2\theta+\cos^2\theta}$

$=\frac{2\sin\theta\cos\theta}{2\cos^2\theta}=\frac{\sin\theta}{\cos\theta}$

$=\tan\theta$

⑥ see online "handout" for $\cos 3x$ derivation.

⑦ a) $\frac{1}{2}[\sin(4x+3x)+\sin(4x-3x)]$

$=\frac{1}{2}[\sin 7x+\sin x]$

b) $2\sin\left(\frac{195^\circ+105^\circ}{2}\right)\cos\left(\frac{195^\circ-105^\circ}{2}\right)$

$=2\sin 150^\circ\cos 45^\circ$

$=2\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}$

⑧ a) $\alpha=12.59^\circ$

$\theta=192.59^\circ$ QIII

$\theta=347.41^\circ$ QIV

b) $0\le 3x\le 6\pi$ (to search)

$3x=\frac{\pi}{2}\Rightarrow x=\frac{\pi}{6}$

$3x=\frac{5\pi}{2}\Rightarrow x=\frac{5\pi}{6}$

$3x=\frac{9\pi}{2}\Rightarrow x=\frac{9\pi}{6}=\frac{3\pi}{2}$

c) $\tan\theta=\frac{1\pm\sqrt{5}}{2}$

$\tan\theta=\frac{1+\sqrt{5}}{2}\Rightarrow\theta=58.28^\circ$ (QI)

$\theta=238.28^\circ$ (QIII)

$\tan\theta=\frac{1-\sqrt{5}}{2}\Rightarrow\alpha=31.72^\circ$ (ref.)

$\theta=148.28^\circ$ (QII)

$\theta=328.28^\circ$ (QIV)

d) $2\sin^2\theta-(\cos^2\theta-\sin^2\theta)=0$

$2\sin^2\theta-\cos^2\theta+\sin^2\theta=0$

$3\sin^2\theta-(1-\sin^2\theta)=0$

$4\sin^2\theta-1=0$

$4\sin^2\theta=1$

$\sin\theta=\pm\frac{1}{2}$

$\theta=30^\circ, 150^\circ, 210^\circ, 330^\circ$