

MAC 1105 PRACTICE EXAM IV (70 pts.)

(10) ① Sketch

$$y = f(x) = x^2 + 2x + 3$$

Show how you get the vertex and plot at least 5 points including the vertex.

(10) ② a) A rectangular feeding pen for cattle is to be made with 500 ft. of fencing and a large barn as one side of the pen. If  $x$  represents the width of the pen, express its area  $A(x)$  in terms of  $x$ .

b) Find the maximum area possible in part a).

(15) ③ Sketch

$$y = f(x) = \frac{x^2}{x^2 - 1}$$

Include all intercepts, table of values and asymptotes.

(10) ④ a) Let  $f(x) = \sqrt{x}$ ,

$$g(x) = \frac{5}{x^2 + 1}$$

Find  $(f \circ g)(x)$  and its domain.

b) Now find different functions  $f, g$  such that

$$H(x) = (x^4 - 2x)^{1/3}$$

can be written in the form

$$H = f \circ g \quad \left( \begin{array}{l} f(x) \neq x \\ g(x) \neq x \end{array} \right)$$

(15) ⑤ a) Let  $y = f(x) = x^2 + 2, x \leq 0$ . Solve for  $f^{-1}(x)$  algebraically.

Include any restrictions.

b) Sketch  $y = g(x) = \sqrt{x+1}$  10 pts. and  $y = g^{-1}(x)$  on the same set of axes.

Label each one.

(5) ⑥ Sketch  $y = \frac{1}{2}(3^{-x})$ .

(5) ⑦ The percent  $p$  of light that passes through  $n$  successive panes of glass is

$$p = 100e^{-0.03n}$$

What percent of light will pass through 20 panes?

# MAC 1105 PRAC. EXAM IV KEY

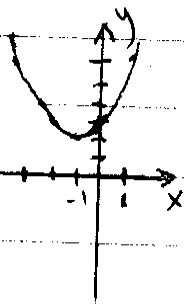
①  $f(x) = x^2 + 2x + 3$

$f(0) = 3$

$x = -\frac{b}{2a} = -1$

$f(-1) = 1 - 2 + 3 = 2$

$f(1) = 6 \quad f(2) = 11$



② a) **BARN**



$A = xy$

$2x + y = 500$

$y = 500 - 2x$

$A = x(500 - 2x)$

$= -2x^2 + 500x$

b)  $x = -\frac{b}{2a} = -\frac{500}{2(-2)} = 125$

$A_{max} = 125(500 - 2(125))$

$= 31,250 \text{ sq. ft.}$

③

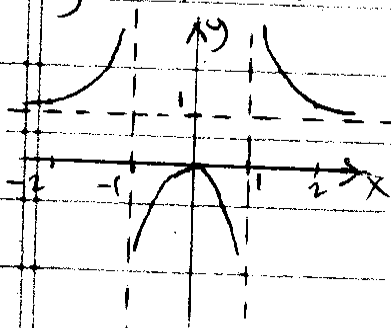
$y = f(x) = \frac{x^2}{(x+1)(x-1)}$

VA:  $x = \pm 1, x = 0$  (x int.)

$f(0) = 0$  (y int.) HA:  $\frac{1x^2}{1x^2} = 1 = y$

$f(2) = \frac{4}{3}$

$f(-2) = \frac{4}{3}$



⑤ a)  $x = y^2 + 2, y \leq 0$

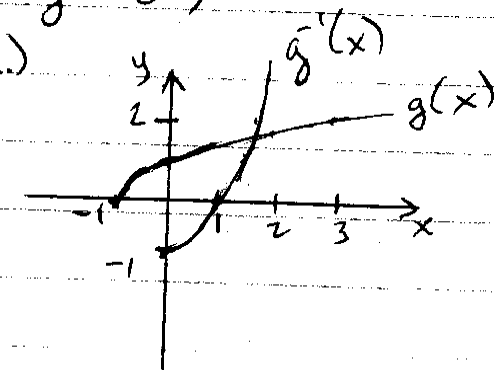
$x - 2 = y^2$

$y = f^{-1}(x) = -\sqrt{x-2}$

(minus, since  $y \leq 0$ )

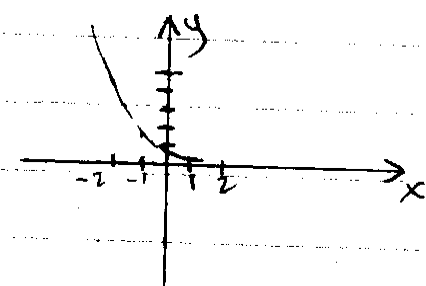
b) x y (approx.)

-1	0
0	1
1	1.4
2	1.7
3	2



⑥

x	y
-2	4.5
-1	1.5
0	0.5
1	.17
2	.06



⑦  $100e^{-0.03(25)} \approx 55 \text{ percent}$

④ a)  $f(g(x)) = f\left(\frac{5}{x^2+1}\right)$

$= \sqrt{\frac{5}{x^2+1}}$

domain: all reals

b)  $f(x) = x^{1/3}$

$g(x) = x^4 - 2x$