The Binomial Theorem

Recall that you can't distribute exponents over terms. For example,

$$(a+b)^2 \neq a^2+b^2 \quad \text{and} \quad (a+b)^3 \neq a^3+b^3.$$

But $(a+b)^0 = 1$, $(a+b)^1 = 1a+1b$, and $(a+b)^2 = a^2+2ab+b^2$.

Also, $(a+b)^3 = 1a^3+3a^2b+3ab^2+b^3$. And so on.

There is a pattern here. The powers on the "a" decrease. b comes into the second term, and the powers on it increase. The coefficient of the second term is the same as the original exponent. To get the coefficient of a term, multiply the coefficient of the previous term by the power of a, and divide by one more than the power on b.

For example, $(a+b)^4 = 1a^4+4a^3b+6a^2b^2+4ab^3+1b^4$.

The 6 is obtained by multiplying the previous 4 by 3, and dividing by 1 more than the exponent 1 on the b. In other words , 12 divided by 2.

Another way to obtain the coefficients is Pascal's triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

Add any two numbers and write the answer below and between them. (Note the symmetric nature of the coefficients.

Exercises: 1) Write out the next 3 rows in Pascal's triangle.

2)Expand $(a+b)^5$

3)Expand $(a+b)^6$

In general, $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \ldots + b^n$.