(70 pts.)

MAC 2233 EXAM II (MWF) MR NADEL SPRING 2010

(10) ① Find the derivative of
$y = f(x) = -3x^2 + 2x$ using the definition of derivative. (the one with "h").

(5) ② If $C(x) = 0.04x^2 + 5.1x + 40$ is the cost in thousands of dollars of producing x units, find the rate of change of cost when $x = 20$.

(10) ③ a) Find y' without Q.R.
$$y = \frac{1}{t} - \frac{2}{t^2} + \frac{3}{t^3}$$

b) Find an equation of the tangent line to
$y = x^2 + \sqrt[3]{x}$ at $x = 1$.

(5) ④ If $H(t) = -16t^2 + 200t$ ft represents the height of an object after t seconds, find the acceleration when $t = 2$ sec

(25) ⑤ a) If $y = u^2 - 3u^{-1}$ and $u = x^3 - 2x$, find $\frac{dy}{dx}$ when $x = -1$.

b) Find y' without Q.R.:
$$y = \frac{3}{(7x^2 + 2x)^{50}}$$

c) Find y': $y = \frac{(x^2+1)^{40}}{2x - 3}$

d) Find $f'(1)$ if
$f(x) = x^2\sqrt{2x+7}$

e) Total cost for q units is $C(q) = 0.2q^2 + q + 900$ dollars. $q(t) = t^2 + 100t$ units are made during t hrs. Find the rate cost changes with respect to time after 2 hrs.

(15) ⑥ a) the total cost of producing q units is $C(q) = 3q^2 + q + 500$ dollars. Use marginal analysis to estimate the cost of manufacturing the 51^{st} unit.

b) Now, compute the actual cost of manufacturing the 51^{st} unit.

c) It's projected that t years from now, the circulation of a newspaper will be
$C(t) = 100t^2 + 400t + 5000$.
Estimate the amount by which the circulation will increase during the next 3 months, using Calculus.
(Use 3 months $= \frac{1}{4}$ year)

SP '10

MAC 2233 EXAM II KEY (MWF)

① $\frac{f(x+h)-f(x)}{h} = \frac{-3(x+h)^2+2(x+h)-(-3x^2+2x)}{h}$

$= \frac{\cancel{-3x^2}-6xh-3h^2+\cancel{2x}+2h+\cancel{3x^2}-\cancel{2x}}{h}$

$= \frac{\cancel{h}(-6x-3h+2)}{\cancel{h}}$

Let $h \to 0$. $f'(x) = -6x+2$

② $C'(x) = 0.08x + 5.1$

$C'(20) = .08(20)+5.1 = 6.7$

③a) $y = t^{-1}-2t^{-2}+3t^{-3}$

$y' = -t^{-2}+4t^{-3}-9t^{-4}$

b) $y = x^2+x^{1/3}$

$y' = 2x+\frac{1}{3}x^{-2/3}$

At $x=1$, $y' = 2+\frac{1}{3} = \frac{7}{3}$

$(1,2)$ $y-2 = \frac{7}{3}(x-1)$

OR $y = \frac{7}{3}x - \frac{1}{3}$

④ $v(t) = H'(t) = -32t+200$

$a(t) = H''(t) = -32$

$H''(2) = -32 \frac{\text{ft/sec}}{\text{sec}}$

⑤a) $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$

$x=-1 \Rightarrow u = -1+2 = 1$

Get $(2u+3u^{-2})(3x^2-2)$

$(2+3)(3-2) = 5(1)$

$= 5$

b) $y = 3(7x^2+2x)^{-50}$

$y' = -150(7x^2+2x)^{-51}(14x+2)$

C.R.

⑤c) $\frac{(2x-3)\,40(x^2+1)^{39}(2x) - (x^2+1)^{40}\cdot 2}{(2x-3)^2}$ C.R.

d) $f(x) = x^2(2x+7)^{1/2}$

$f'(x) = x^2\cdot\frac{1}{\cancel{2}}(2x+7)^{-1/2}\cdot\cancel{2} + (2x+7)^{1/2}\cdot 2x$

$f'(1) = \frac{1}{3}+3\cdot 2 = 6\frac{1}{3}$ OR $\frac{19}{3}$

e) $\frac{dC}{dq}\cdot\frac{dq}{dt} = \frac{dC}{dt}$

$(0.4q+1)(2t+100)$

$t=2 \Rightarrow q = 4+200 = 204$

$\frac{dC}{dt} = [0.4(204)+1](2(2)+100)$

$= (82.6)(104) = 8590.4$

⑥a) $C'(q) = 6q+1$

$C'(50) = 6(50)+1 = 301$

b) $C(51)-C(50)$

$= 8354 - 8{,}050 = 304$

c) $\frac{\Delta C}{\Delta t} \approx C'(t)$

$\Delta C \approx C'(t)\cdot\Delta t$

Currently $t=0$.

$\Delta t = \frac{1}{4}$ yr.

$C'(t) = 200t+400$

$\Delta C \approx [200(0)+400]\cdot\frac{1}{4}$

$= 100$