(65 pts.)

MAC 2233 EXAM I MR. NADEL FALL 2011

(15) ① a) $\int \left(e^{-5x} + \frac{9}{x^2} + 2\right) dx$

b) $\int \left(\frac{x^3 - 3x^2 + 5}{x}\right) dx$

c) The marginal cost of a commodity is $6x^2 - 2x + 5$, when x is the level of production. If it costs \$5 to produce 1 unit, what's the total cost of producing 5 units?

(15) ② a) $\int x\sqrt{x+2}\, dx$

b) $\int x\sqrt[3]{x^2+2}\, dx$

c) The population P of a certain town is currently 13,000. If it's increasing at the rate of $200e^{.1t}$ people per year, t years from now, find a formula for $P(t)$.

(20) ③ a) $\int_2^3 \left(\frac{4}{x} + \frac{1}{x^2}\right) dx$

b) $\int_0^1 (2x+1)^3\, dx$

c) $\int_1^2 \frac{x^3}{(x^4+1)^2}\, dx$

d) The population of a town will be increasing at the rate of $3 + x^{4/5}$ $\frac{\text{people}}{\text{month}}$ x months from now. By how much will it increase over the next 6 months?

(10) ④ Find the area between $y = 6 - x^2$ and $y = x$.

(5) ⑤ Find the average value of $y = f(x) = \sqrt{x-1}$ on $5 \le x \le 10$

MAC 2233 EXAM II KEY (F '11)

① a) $\int (e^{-5x} + 9x^{-2} + 2)\,dx$

$= \frac{e^{-5x}}{-5} + \frac{9x^{-1}}{-1} + 2x + C$

b) $\int (x^2 - 3x + \frac{5}{x^2})\,dx$

$= \frac{x^3}{3} - \frac{3x^2}{2} + 5\ln|x| + C$

c) $C(x) = 2x^3 - x^2 + 5x + K$

$C(1) = 2 - 1 + 5 + K = 5 \Rightarrow K = -1$

$C(5) = 2(125) - 25 + 25 - 1 = \$249$

② a) $u = x + 2 \quad u - 2 = x \quad du = dx$

$\int (u-2)u^{1/2}\,du = \int (u^{3/2} - 2u^{1/2})\,du$

$= \frac{2}{5}u^{5/2} - 2\left(\frac{2}{3}\right)u^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

b) $u = x^2 + 2 \quad du = 2x\,dx \quad \frac{1}{2}du = x\,dx$

$\frac{1}{2}\int u^{1/3}\,du = \frac{1}{2}\cdot\frac{3}{4}u^{4/3} + C$

$= \frac{3}{8}(x^2+2)^{4/3} + C$

c) $P(t) = \frac{200e^{.10t}}{.10} + C$

$= 2000e^{.10t} + C$

$P(0) = 13000 = 2000 + C$

$C = 11{,}000 \quad P(t) = 2000e^{.10t} + 11{,}000$

③ a) $(4\ln|x| - x^{-1})\Big|_2^3$

$= 4\ln 3 - \frac{1}{3} - (4\ln 2 - \frac{1}{2}) \approx 1.789$

b) $\frac{(2x+1)^4}{8}\Big|_0^1 = \frac{3^4 - 1^4}{8} = \frac{80}{8} = 10$

(or let $u = 2x+1$)

c) $u = x^4 + 1 \quad du = 4x^3\,dx$

$\frac{1}{4}du = x^3\,dx$

$\frac{1}{4}\int_2^{17} u^{-2}\,du = \frac{1}{4}\frac{u^{-1}}{-1}\Big|_2^{17}$

$= -\frac{1}{4}\left(\frac{1}{17} - \frac{1}{2}\right) = \frac{15}{136}$

d) $\int_0^6 (3 + x^{4/5})\,dx = \left(3x + \frac{5}{9}x^{9/5}\right)\Big|_0^6$

$= 18 + \frac{5}{9}(6)^{9/5} \approx 32$ people

④ $6 - x^2 = x$

$0 = x^2 + x - 6$

$(x+3)(x-2) = 0$

$x = -3, 2$

$\int_{-3}^{2} (6 - x^2 - x)\,dx$ (top − bottom)

$= \left(6x - \frac{x^3}{3} - \frac{x^2}{2}\right)\Big|_{-3}^{2}$

$= \frac{125}{6}$ OR $20\frac{5}{6}$

⑤ $\frac{1}{5}\int_5^{10} (x-1)^{1/2}\,dx = \frac{1}{5}\cdot\frac{2}{3}(x-1)^{3/2}\Big|_5^{10}$

$= \frac{2}{15}\left[9^{3/2} - 4^{3/2}\right] = \frac{38}{15}$