MAC 2233 EXAM VI - MR. NADEL FALL 2011

(20) 1 a) Find the future value of a continuous income stream with a flow rate of \$5,000 per year at 5% compounded continuously for 10 years

b) the demand and supply functions are $D(q) = \frac{20}{q+1}$ and $S(q) = q+2$. Find the equilibrium price and quantity

c) Now find the consumers' surplus.

d) What's the producers' surplus in this example?

(15) 2 $f(x,y) = \frac{3x^2+4y}{x+y}$ $g(x,y) = \frac{e^{xy}}{3+e^{xy}}$

(next column)

a) Find domain of f.

b) Find domain of g

c) Compute $g(\ln 2, 2)$ on calculator.

(15) 3 a) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$z = \frac{x^2 y^3}{x^2 y + 2}$$

b) If $f(x,y) = (x + 3xy - y)^4$ find f_{xy}

c) Daily output is $Q(K,L) = 10K^{1/3} L^{1/2}$ units. Use a partial derivative to estimate the change in output if L goes from 625 to 626 while K remains constant at 216.

(15) 4 Suppose your productivity during the work day (indexed by a number from 0 to 100) is given by

$$P(x,y) = -x^2 - 9y^2 + 16x + 18y + 27$$

where x is the no. of hours you sleep, and y the no. of hours you exercise, find the number of hours of sleeping and exercising to maximize your productivity. Also what is the productivity number?

Justify <u>and</u> show all work.

MAC 2233 (F'11) EXAM VI KEY (MWF)

① a) $e^{.05(10)}\int_0^{10} 5000\, e^{-.05t}\,dt$

$= \dfrac{e^{.5}(5000)}{-.05} e^{-.05t}\Big|_0^{10}$

$= \dfrac{e^{.5}(5000)}{-.05}\left(e^{-.5}-1\right)$

$= \$64,872.13$

b) $\dfrac{20}{q+1} = q+2 \quad 20=(q+2)(q+1)$

$20 = q^2+3q+2$

$0 = q^2+3q-18 = (q-3)(q+6)$

$q = 3, \cancel{-6} \Rightarrow p = 5$

① c) $\int_0^3 \dfrac{20}{q+1}\,dq - (3)(5)$

$20\ln(q+1)\Big|_0^3 - 15 = \$12.73$

d) $15-\int_0^3 (q+2)\,dq = 15-\left(\frac{q^2}{2}+2q\right)\Big|_0^3$

$= 15-\left(\frac{9}{2}+6\right) = \$4.50$

② a) $x+y \neq 0$ or $y \neq -x$

b) $3+e^{xy} \neq 0$, so all reals.

c) $\dfrac{e^{(\ln 2)(2)}}{3+e^{(\ln 2)(2)}} = \dfrac{4}{3+4} = \dfrac{4}{7}$

③ a) $\dfrac{(x^2y+2)(2xy^3)-x^2y^3(2xy)}{(x^2y+2)^2}$

b) $f_x = 4(x+3xy-y)^3(1+3y)$ C.R.

$f_{xy} = 4(x+3xy-y)^3(3)$
$\quad + (1+3y)12(x+3xy-y)^2(3x-1)$ C.R.

Using P.R.

c) $Q_L = 5K^{1/3}L^{-1/2} = \dfrac{5K^{1/3}}{L^{1/2}}$

$= \dfrac{5(216)^{1/3}}{(625)^{1/2}} = \dfrac{30}{25} = 1.2$

④ $P_x = -2x+16 = 0 \Rightarrow x = 8$

$P_y = -18y+18 = 0 \Rightarrow y = 1$

$P_{xx} = -2 \quad P_{yy} = -18$

$P_{xy} = P_{yx} = 0$

$D = (-2)(-18)-0^2 = 36 > 0$

$P_{xx} = -2 < 0 \Rightarrow$ rel. max.

$P = -64-9+128+18+27$

$= 100$ is maximum #

8 hrs. of sleep and
1 hr. of exercise.