

(10) ① A rectangular picnic area, 6000 square yards, is to be fenced off on 3 sides not adjacent to the highway. What's the least amount of fencing needed? Use Calculus.

(15) ② a) Suppose \$3,000 is invested at an annual rate of 11% for 5 years. Compute the balance if the interest is compounded monthly.

b) Repeat a) but compounded continuously.

c) A radioactive substance decays exponentially. If 500 grams of the substance were present initially, and 300 grams are present 75 years later, how many grams will be present 300 years after the initial time?

(Do not use logarithms.)

(15) ③ a) Find  $\ln\left(\frac{a}{b\sqrt{c}}\right)$  if  $\ln a = -2$ ,  $\ln b = 5$ ,  $\ln c = 4$ .

b) A bank doubles your money in 12 years, compounded continuously. Find the annual rate.

c) Solve  $4 = 2 + 3e^{-5x}$  for  $x$ .

Give calculator answer.

(15) ④ a) Find  $f'(x)$  if

$$f(x) = (1 + 2e^{5x})^3$$

b) Find  $y'$  if  $y = x^3 \ln(x^2) + e^6$

c) Use logarithmic differentiation.

$$y = (x-3)^4 / (2x+1)^7$$

(15) ⑤ a) Sketch  $y = 4 - 3e^{-x}$

b) The value <sup>in dollars</sup> of a machine is

$$V(t) = 4800e^{-t/5} + 400.$$

Find its value after 15 years.

c) Now, find the salvage value.

①

$$\begin{aligned}
 xy &= 6000 \\
 y &= \frac{6000}{x} \\
 F &= 2y + x \\
 &= 2\left(\frac{6000}{x}\right) + x \\
 &= 12,000x^{-1} + x \\
 F' &= -12,000x^{-2} + 1 = 0 \\
 1 &= \frac{12,000}{x^2}
 \end{aligned}$$

$$x = \sqrt{12,000} \approx 109.54$$

$$y \approx 54.77$$

$$F \approx 219.08 \text{ or } 219.09$$

$$\textcircled{2} \text{ a) } 3000 \left(1 + \frac{11}{12}\right)^{5(12)} = \$5186.75$$

$$\begin{aligned}
 \text{b) } B &= Pe^{rt} = 3000e^{11(5)} \\
 &= \$5199.76
 \end{aligned}$$

$$\text{c) } Q = Q_0 e^{-kt}$$

$$300 = 500 e^{-k(75)}$$

$$\frac{3}{5} = e^{-75k}$$

$$\begin{aligned}
 Q &= 500 e^{-k(300)} = 500 (e^{-75k})^4 \\
 &= 500 \left(\frac{3}{5}\right)^4 \\
 &= 64.8 \text{ g}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \text{ a) } \ln a - \ln b \sqrt{c} &= \ln a - (\ln b + \frac{1}{2} \ln c) \\
 &= \ln a - \ln b - \frac{1}{2} \ln c \\
 &= -2 - 5 - \frac{1}{2}(4) = -9
 \end{aligned}$$

$$\text{b) } B = Pe^{rt}$$

$$2P = Pe^{-r(12)} \quad \ln 2 = (2r)$$

$$r = \frac{\ln 2}{12} = 5.78\%$$

$$\text{c) } 2 = 3e^{-5x} \quad \frac{2}{3} = e^{-5x} \quad \ln\left(\frac{2}{3}\right) = -5x$$

$$x = \frac{\ln\left(\frac{2}{3}\right)}{-5} \approx .0811$$

$$\textcircled{4} \text{ a) } 3(1 + 2e^{5x})^2 (10e^{5x}) \leftarrow \text{C.R.}$$

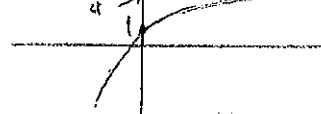
$$\text{b) } x^3 \left(\frac{1}{x^2}\right) \cdot 2x + \ln(x^2) \cdot 3x^2 + 0$$

$$\text{c) } \ln y = 4 \ln(x-3) - 7 \ln(2x+1)$$

$$\frac{y'}{y} = \left[ \frac{4}{x-3} - \frac{7(2)}{2x+1} \right] \leftarrow \text{C.R.}$$

$$y' = y \cdot \left[ \frac{4}{x-3} - \frac{14}{2x+1} \right]$$

⑤ a)



$$\text{b) } 4800 e^{-15/5} + 400 = \$638.98$$

$$\text{c) } \$400$$