

(10) ① Find the domain, intervals of increase, decrease, and concavity for $f(x) = (x+1)^{2/3}$. Don't sketch.

(10) ② Sketch $y = f(x) = x^3 - 3x + 1$. Include intervals of increase, decrease, concavity, & y intercept.

(15) ③ Sketch $y = f(x) = \frac{2x^2 - 8}{x^2 - 16}$

$$\text{Hint: } f'(x) = -\frac{48x}{(x^2-16)^2} \quad f''(x) = \frac{48(16+3x^2)}{(x^2-16)^3}$$

(10) ④ Find the absolute maximum and minimum values of

$$f(x) = 2x^3 - 3x^2 - 12x + 15 \text{ on } 0 \leq x \leq 3.$$

(10) ⑤ The total cost in dollars of manufacturing q units of a commodity is

$$C(q) = 5q^2 + 6q + 125$$

a) At what level of production is the average cost smallest?

b) Where is average cost = marginal cost?

(5) ⑥ If $P(x) = -400(x-2)(x-15)$ is profit, find the maximum profit. (Use Calculus).

(MWF) (60 pts.)
 MAC 2233 (F' II) EXAM III KEY

② $f(x) = x^3 - 3x + 1$ $f(0) = 1$

$f'(x) = 3x^2 - 3 = 0 \Rightarrow x = \pm 1$

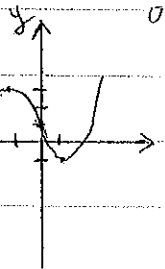
inc. | dec. | inc.

-1 | 1

$f(-1) = 3$ $f(1) = -1$

$f''(x) = 6x$

c.d. | c.u.



① $f(x) = (x+1)^{2/3}$ domain: all reals

$f'(x) = \frac{2}{3}(x+1)^{-1/3} = \frac{2}{3(x+1)^{1/3}}$

dec. | inc.

-1

$f''(x) = -\frac{2}{9}(x+1)^{-4/3} = \frac{-2}{9(x+1)^{4/3}} < 0$

f is concave down.

③ $x = \pm 2$ (x int.) $f(0) = \frac{1}{2}$ (y int.)

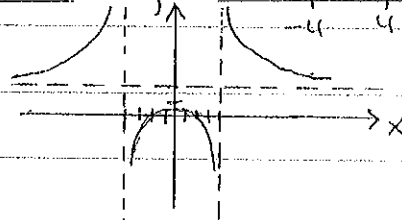
VA: $x = \pm 4$

HA: $y = 2$

inc. | dec.

0

c.u. | c.d. | c.u.



④ $f'(x) = 6x^2 - 6x - 12$

$= 6(x-2)(x+1) = 0 \Rightarrow x = 2, -1$

$f(0) = 15$, $f(2) = -5$, $f(3) = 6$

↑
max

↑
min

⑤ a) $A(q) = \frac{5q^2 + 6q + 125}{q} = 5q + 6 + 125q^{-1}$

$A'(q) = 5 - 125q^{-2} = 0$

$5 = \frac{125}{q^2}$, $q^2 = 25$, $q = 5$

b) $5q + \frac{125}{q} + 125q^{-1} = 10q + \frac{125}{q}$

$\frac{125}{q} = 5q \Rightarrow q^2 = 25$, $q = 5$

⑥ $P(x) = -400(x^2 - 17x + 30)$

$P'(x) = -400(2x - 17) = 0$

$x = \frac{17}{2} = 8.5$

$P_{\max} = P(8.5) = -400(8.5-2)(8.5-15) = 16,900$