(65 pts.)

MAC 2233 EXAM V MR. NADEL SPRING 2010

(15) ① a) $\int \left(\frac{3}{x} + \frac{3}{x^2} + \frac{3}{e^x}\right) dx$

b) $\int \sqrt{x}\left(8x - \frac{7}{x}\right) dx$

c) The population of a town will be increasing at the rate of $3 + 5x^{3/2}$ people per month, x months from now. Currently the population is 7,000. Find the population in 9 months.

(15) ② a) $\int x^2 \sqrt{x^3+9}\, dx$

b) $\int x\sqrt{x-2}\, dx$

c) Water flows into a tank at the rate of $(2t+3)^{1/2}$ ft^3/min. If the tank is empty when $t=0$, how much water does it contain 4 min. later? (Answer to 2 dec. places.)

(20) ③ a) $\int_1^2 \left(7 - \frac{2}{x}\right) dx$

b) $\int_0^1 (x^3 - 3x^2 + e^{-2x})\, dx$

c) $\int_0^1 \frac{x\, dx}{(2x^2+3)^2}$

(next column)

③ d) It's estimated that t days from now a crop will increase at the rate of

$0.3t^2 + 0.6t + 1$ $\frac{\text{bushels}}{\text{day}}$.

By how much will the crop's value increase during the next 7 days, if the price stays fixed at \$6 per bushel.

(10) ④ Find the <u>area</u> in the <u>first</u> quadrant between $y = x^2$ and $y = x^4$.

(5) ⑤ Find the <u>average value</u> of the function

$$y = f(x) = \frac{3}{x+1}$$

over the interval $1 \le x \le 5$.

(MWF)

MAC 2233 EXAM I KEY (SP '10)

① a) $\int\left(\frac{3}{x}+3x^{-2}+3e^{-x}\right)dx$

$= 3\ln|x| + \frac{3x^{-1}}{-1} + \frac{3e^{-x}}{-1} + C$

b) $\int x^{1/2}\left(8x-\frac{7}{x}\right)dx$

$= \int(8x^{3/2}-7x^{-1/2})\,dx$

$= 8\cdot\frac{2}{5}x^{5/2} - 7(2)x^{1/2} + C$

$= \frac{16}{5}x^{5/2} - 14x^{1/2} + C$

c) $P = \int(3+5x^{3/2})\,dx$

$= 3x + 2x^{5/2} + C$

$C = 7000$

$P = 3x + 2x^{5/2} + 7000$

When $x=9$, $P = 7513$

② a) $u = x^3+9$, $du = 3x^2dx$, $\frac{1}{3}du = x^2dx$

$\frac{1}{3}\int u^{1/2}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}(x^3+9)^{3/2} + C$

b) $u = x-2 \Rightarrow u+2 = x$, $du = dx$

$\int(u+2)u^{1/2}\,du = \int(u^{3/2}+2u^{1/2})\,du$

$= \frac{2}{5}u^{5/2} + 2\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + C$

c) $\int(2t+3)^{1/2}\,dt$

$= \left(\frac{1}{2}\right)\cdot\frac{2}{3}(2t+3)^{3/2} + C$

↑ to balance C.R.

$= \frac{1}{3}(2t+3)^{3/2} + C$

$t=0 \rightarrow \frac{1}{3}(3)^{3/2} + C = 0$

$\rightarrow C = -\frac{1}{3}(3)^{3/2}$

$W(t) = \frac{1}{3}(2t+3)^{3/2} - \frac{1}{3}(3)^{3/2}$

$W(4) = \frac{1}{3}(11)^{3/2} - \frac{1}{3}(3)^{3/2}$

≈ 10.43

③ a) $(7x - 2\ln|x|)\Big|_1^2$

$= 14 - 2\ln 2 - (7 - 2\ln 1)$

≈ 5.6137

b) $\left(\frac{x^4}{4} - x^3 + \frac{e^{-2x}}{-2}\right)\Big|_0^1$

$= \frac{1}{4} - 1 + \frac{e^{-2}}{-2} - \left(-\frac{1}{2}\right) \approx -.318$

c) $u = 2x^2+3$, $du = 4x\,dx$, $\frac{1}{4}du = x\,dx$

$\frac{1}{4}\int_3^5 u^{-2}\,du = -\frac{1}{4}u^{-1}\Big|_3^5$

$= -\frac{1}{20} + \frac{1}{12} = \frac{2}{60} = \frac{1}{30}$

d) $6\int_0^7(0.3t^2+0.6t+1)\,dt$

$= 6(.1t^3+0.3t^2+t)\Big|_0^7$

$= 6(.1(7)^3+0.3(7)^2+7) = \$336$

④ $\int_0^1(x^2-x^4)\,dx = \left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$

$= \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

⑤ $\frac{1}{4}\int_1^5 \frac{3}{x+1}\,dx$

$= \frac{3}{4}\ln|x+1|\Big|_1^5$

$= \frac{3}{4}(\ln 6 - \ln 2) \approx .824$

Picture for ④