(70 pts.)

MAC 2233 EXAM II (MWF) MR. NADEL FALL 2011

(10) ① Find the derivative of $f(x) = x^2 - 3x$ using the definition.

(5) ② If $P(x) = -200x^2 + 7{,}000x - 10{,}000$ is profit, where x = selling price, find the optimal selling price. (Use short rule.)

(10) ③ a) Find equation of the tangent line to $y = x^5 - 2x^3 + 3x + 17$ at $x = 1$.

b) Find y' if $y = (2x^3 - 4)(x^5 + 2x)$ using the Product Rule.

(5) ④ A worker produces $Q(t) = -t^3 + 8t^2 + 15t$ units, t hrs. after 8 AM. If $R(t) = Q'(t)$, at what rate is $R(t)$ changing at 10 AM?

(25) ⑤ a) Find $\frac{dy}{dx}$ if $y = u^4 - 3u^2 + 9$, $u = x^3 - 2x + 1$.

b) Find y' if $y = \dfrac{(2x^2 - 3)^5}{(x^2 + 1)^2}$

c) Find y' if

$$y = \sqrt{1 - \frac{1}{2x}}$$

without Q.R. or P.R.

(next column)

⑤ d) Find y' if

$$y = \frac{2}{(x^8 - 1)^3}$$

without Q.R. or P.R.

e) Demand for coffee is $D(p) = 2967p^{-2}$ pounds per week when price is p dollars. t weeks from now the price will be

$$p(t) = 0.02t^2 + 0.1t + 6$$

dollars per pound. At what rate will demand for coffee be changing with respect to time 9 weeks from now?

(15) ⑥ a) Suppose total cost in dollars of manufacturing q units is

$$C(q) = 3q^2 + 8q + 9.$$

Use marginal analysis to estimate the cost of manufacturing the 41st unit.

b) Compute the actual cost of manufacturing the 41st unit.

c) Daily output of a factory is $Q(K) = 800K^{1/2}$ where K is capital investment in units of \$1,000. Current investment is \$625,000. Estimate the effect an additional investment of \$700 will have on daily output. (Use Calculus.)

MAC 2233 EXAM II KEY (F'11) MWF

① $\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$

$= \frac{\cancel{x^2}+2xh+h^2-\cancel{3x}-3h-\cancel{x^2}+\cancel{3x}}{h}$

$= \frac{\cancel{h}(2x+h-3)}{\cancel{h}}$, Let $h \to 0$.

$f'(x) = 2x-3$.

② $-400x + 7{,}000 = 0$

$7000 = 400x \Rightarrow x = 17.5$

③ a) $y' = 5x^4 - 6x^2 + 3$

$m_{tan} = 5-6+3 = 2$ $(1, 19)$

$y - 19 = 2(x-1)$

OR $y = 2x + 17$

b) $(2x^3-4)(5x^4+2) + (x^5+2x)(6x^2)$

④ $R(t) = Q'(t) = -3t^2+16t+15$

$R'(t) = -6t+16$

10AM $(t=2)$, $R'(2) = -12+16$

$= 4 \frac{\text{units/hr}}{\text{hour}}$

⑤ a) $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx} = (4u^3-6u)(3x^2-2)$

$= (4(x^3-2x+1)^3 - 6(x^3-2x+1))(3x^2-2)$

b) $\frac{(x^2+1)^2 5(2x^2-3)^4(4x) - (2x^2-3)^5 2(x^2+1)(2x)}{(x^2+1)^4}$ ← C.R. ← C.R.

c) $y = (1-\frac{1}{2}x^{-1})^{1/2}$

$y' = \frac{1}{2}(1-\frac{1}{2}x^{-1})^{-1/2}(\frac{1}{2}x^{-2})$ ← C.R.

d) $y = 2(x^8-1)^{-3}$

$y' = -6(x^8-1)^{-4}(8x^7)$ ← C.R.

(next column)

⑤ e) $\frac{dD}{dt} = \frac{dD}{dp}\cdot\frac{dp}{dt}$

$= (-5934p^{-3})(.04t+0.1)$

$t=9 \Rightarrow p = 8.52$

$\frac{-5934}{(8.52)^3}(.04(9)+.1)$

$= -4.41$ pounds/wk.

⑥ a) $C'(q) = 6q+8$

$C'(40) = 6(40)+8 = \$248$ (per unit)

b) $C(41) - C(40)$

$= \$5380 - \$5129 = \$251$

c) $Q'(K) = 400K^{-1/2}$

$\Delta Q \approx Q'(K)\,\Delta K$

$\frac{400}{\sqrt{625}}(.7) = 11.2$

Note: \$625,000 means $K = 625$ and \$700 means $\Delta K = .7$