(MWF) (60 pts.)

MAC 2233 EXAM III MR. NADEL SPRING 2010

(10) ① Find the domain, intervals of increase, decrease, and concavity for $f(x) = 3(x-1)^{1/3} - 2$.

(10) ② Sketch $y = f(x) = 2x^3 + 3x^2 + 1$

Include intervals of increase, decrease, concavity and y intercept.

(15) ③ Sketch $y = f(x) = \dfrac{x^2 - x - 2}{x^2 - 2x + 1} = \dfrac{(x-2)(x+1)}{(x-1)^2}$

Hint: $f'(x) = \dfrac{5-x}{(x-1)^3}$ $\quad f''(x) = \dfrac{2(x-7)}{(x-1)^4}$

A graph may cross its horizontal asymptote.

You will get some fractional y values.

(10) ④ Find the absolute maximum and minimum values of $y = f(x) = x^4 - 2x^3 + 3$ on $-2 \le x \le 3$.

(10) ⑤ The total cost in dollars of manufacturing q units of a commodity is

$$C(q) = 4q^2 + 2q + 64$$

a) At what level of production is the average cost the smallest?

b) Where is average cost = marginal cost?

(5) ⑥ If $D(p) = 200 - 5p$ is the number of units demanded at a price p, in dollars, find the price where expenditure is maximum, and also the maximum expenditure. Use Calculus.

Hint: First, write an equation for expenditure, $E(p)$.

(MWF)

MAC 2233 EXAM III KEY (SP'10)

① $f(x) = 3(x-1)^{1/3} - 2$

domain: all reals

$f'(x) = (x-1)^{-2/3} = \frac{1}{(x-1)^{2/3}} > 0$

f increasing.

$f''(x) = -\frac{2}{3}(x-1)^{-5/3} = \frac{-2}{(x-1)^{5/3}}$

c.u. | c.d.

1

② $y = f(x) = 2x^3 + 3x^2 + 1$

$f'(x) = 6x^2 + 6x = 0$

$6x(x+1) = 0 \Rightarrow x = 0, -1$

inc. | dec. | inc.

-1 0

$f(0) = 1$, $f(-1) = 2$

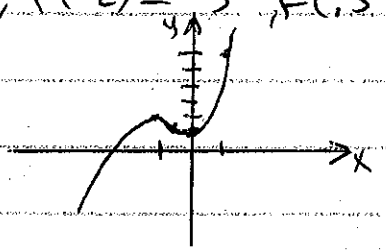
$f''(x) = 12x + 6 = 0 \Rightarrow x = -\frac{1}{2}$

c.d. | c.u.

$-\frac{1}{2}$

$f(-\frac{1}{2}) = 1.5$

$f(1) = 6$, $f(-2) = -3$, $f(.5) = 2$



③ $f(0) = -2 =$ y int.

x int. $= 2, -1$ VA: $x = 1$

HA: $y = 1$

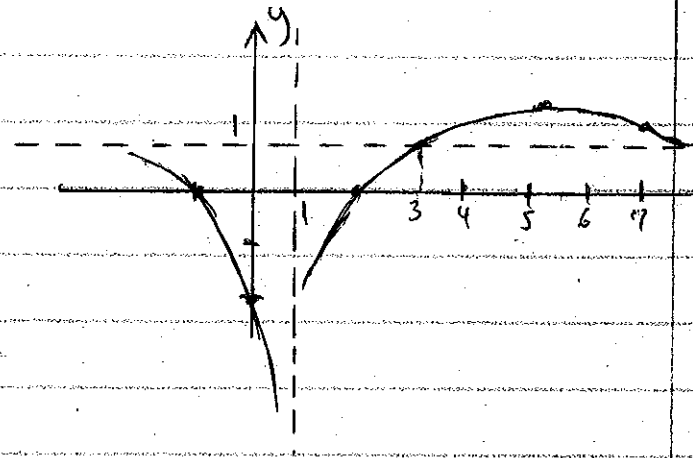
dec. | inc. | dec

1 5

c.d. | c.u.

7

$f(5) = 1.125$ $f(7) \approx 1.11$



④ $f(x) = x^4 - 2x^3 + 3$ $-2 \le x \le 3$

$f'(x) = 4x^3 - 6x^2 = 0$

$2x^2(2x-3) = 0 \Rightarrow x = 0, \frac{3}{2}$

$f(-2) = 35$ (max) $f(0) = 3$, $f(1.5) = 1.3125$ (min.)

$f(3) = 30$

⑤ a) $A(q) = \frac{4q^2 + 2q + 64}{q} = 4q + 2 + 64q^{-1}$

$= 0$

$4 = \frac{64}{q^2} \Rightarrow q^2 = 16 \Rightarrow q = 4$

b) $4q + 2 + 64q^{-1} = 8q + 2$

$64q^{-1} = 4q \Rightarrow 64 = 4q^2$

$q^2 = 16 \Rightarrow q = 4$

⑥ $E(p) = p(200 - 5p)$

$= 200p - 5p^2$

$E' = 200 - 100p = 0$

$\Rightarrow p = \$20$

$E_{max} = 20(200 - 100)$

$= \$2000$