MAC 2312 EXAM IV MR. NADEL FALL 2010

(5) ① Write the sequence

$$\left\{\frac{1}{2}, \frac{4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}, \ldots\right\}$$

in bracket notation.

(5) ② Find the sum of

$$1 - \frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \ldots$$

(10) ③ a) Find an expression for the n^{th} partial sum, S_n, of

$$\sum_{K=1}^{\infty}\left(\frac{1}{2K-1} - \frac{1}{2K+1}\right)$$

in closed form.

b) Find the sum of the series in a).

(45) ④ Test the following series to determine if they converge or diverge. Show all work and give complete explanations to receive credit.

a) Use just the Ratio Test on $\sum_{K=1}^{\infty} \frac{1}{\sqrt{K}}$

What conclusion can be drawn?

b) $-1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \ldots$

c) $\sum_{K=1}^{\infty} \frac{1}{K^{.99}}$

d) $\sum_{K=1}^{\infty} \frac{1}{K\sqrt{K}}$

e) $\sum_{K=1}^{\infty} \left(\frac{K}{2K+100}\right)^K$ Use Root Test

f) $\sum_{K=1}^{\infty} \frac{\sqrt{K}}{K^2+1}$ (Comparison Test)

g) $\sum_{K=2}^{\infty} (-1)^K \frac{K}{\ln K}$

h) $\sum_{K=1}^{\infty} \frac{10^K}{K!}$

i) $\sum_{K=1}^{\infty} \frac{K+1}{K^2+2K}$

(5) ⑤ Find the sum of

$$\sum_{K=1}^{\infty} (-1)^K \frac{1}{3^K \cdot K!}$$

to within .00005.

(5) ⑥ Classify as absolutely convergent, conditionally convergent, or divergent. Explain.

$$\sum_{K=1}^{\infty} \frac{(-1)^K K^2}{e^K}$$

(5) ⑦ Prove $\lim_{n\to\infty} \sqrt[n]{n} = 1$.

MAC 2312 EXAM IV KEY (F'10)

① $\left\{\dfrac{n^2}{n+1}\right\}_{n=1}^{\infty}$

② $\dfrac{1}{1-\left(-\frac{2}{5}\right)} = \dfrac{1}{1+\frac{2}{5}} = \dfrac{1}{\left(\frac{7}{5}\right)} = \dfrac{5}{7}$

③ a) $S_n = \left(1-\cancel{\tfrac{1}{3}}\right)+\left(\cancel{\tfrac{1}{3}}-\cancel{\tfrac{1}{5}}\right)+\dots+\left(\cancel{\tfrac{1}{2n-1}}-\dfrac{1}{2n+1}\right)$

$= 1-\dfrac{1}{2n+1}$

b) $\lim\limits_{n\to\infty}\left(1-\dfrac{1}{2n+1}\right)=1$

④ a) $\lim\limits_{k\to\infty}\dfrac{\sqrt{k}}{\sqrt{k+1}} = \lim\limits_{k\to\infty}\sqrt{\dfrac{k}{k+1}}$

$=1$

No conclusion by Ratio Test.

b) $a_k = \dfrac{1}{k}$ $\quad a_1 \geq a_2 \geq a_3 \geq \dots$

$\lim\limits_{k\to\infty} a_k = 0$

Series converges by AST.

c) $p = .99 \leq 1.$

Series diverges by p-series test.

d) $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^{3/2}}$

$p=\frac{3}{2}>1$

Series converges by p-series test.

e) $\lim\limits_{k\to\infty}\dfrac{1k}{2k+100}=\dfrac{1}{2}<1.$

Series converges by the Root test.

f) $\displaystyle\sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^2+1} \leq \sum_{k=1}^{\infty}\frac{\sqrt{k}}{k^2}$

$\displaystyle = \sum_{k=1}^{\infty}\frac{1}{k^{3/2}}$

Right side converges ($p=\frac{3}{2}>1$) so leftside converges by the Comparison Test.

g) $\lim\limits_{k\to\infty}\dfrac{k}{\ln k} = \lim\limits_{k\to\infty}\dfrac{1}{\left(\frac{1}{k}\right)}$ (by L.R.)

$=\lim\limits_{k\to\infty} k = \infty$

Series diverges by Div. Test.

h) $\lim\limits_{k\to\infty}\dfrac{10^{k+1}}{(k+1)!}\cdot\dfrac{k!}{10^k}$

$=\lim\limits_{k\to\infty}\dfrac{10}{k+1} = 0<1$

Series converges by Ratio Test.

i) $\displaystyle\int_1^{\infty}\frac{x+1}{x^2+2x}\,dx = \lim_{c\to\infty}\int_1^c\frac{x+1}{x^2+2x}\,dx$

$u=x^2+2x$, $du=(2x+2)\,dx$

$\displaystyle\lim_{c\to\infty}\frac{1}{2}\int_3^{c^2+2c}\frac{1}{u}\,du = \lim_{c\to\infty}\frac{1}{2}\ln|u|\Big|_3^{c^2+2c}$

$=\lim\limits_{c\to\infty}\frac{1}{2}\left(\ln|c^2+2c|-\ln 3\right)=\infty$

diverges by Integral Test.

⑤ $|s-s_n| \leq a_{n+1} < .00005$

$\dfrac{1}{3^{n+1}(n+1)!} < .00005$, $\quad n=4$

$-\frac{1}{3}+\frac{1}{18}-\frac{1}{162}+\frac{1}{1944}\approx -.2834$

⑥ $\lim\limits_{k\to\infty}\left|\dfrac{(k+1)^2}{e^{k+1}}\cdot\dfrac{e^k}{k^2}\right| = \dfrac{1}{e}<1$

Series converges absolutely (Ratio Test)

⑦ See notes