

Chap. 12 Practice Problems

- ① a) Convert $(2, \frac{2\pi}{3})$ to rectangular coordinates.
b) Determine all primary representations in polar coordinates of $(\sqrt{3}, 1)$.
- ② Show that $x^2 - y^2 = 1$ has polar equation $r^2 = \sec 2\theta$
- ③ Sketch $r = \sin 2\theta$
- ④ Sketch $r = 1 + 2 \sin \theta$
- ⑤ Find the area of the region bounded by one petal of $r = \cos 2\theta$.
- ⑥ Find the area bounded by $r = 2(1 + \sin \theta)$

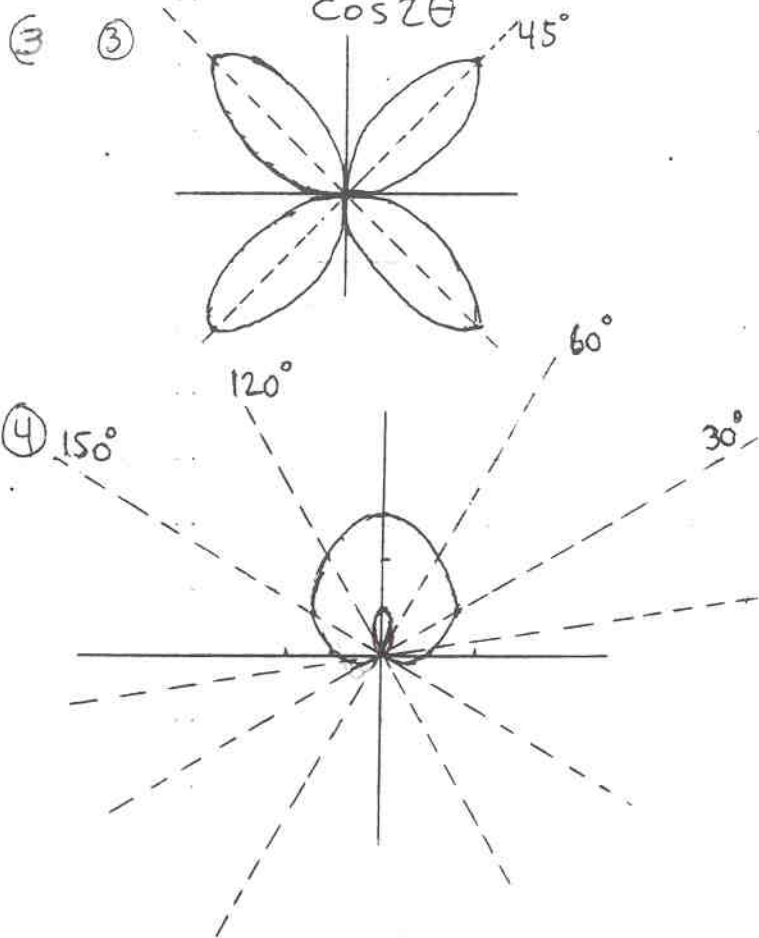
Chap. 12 Solutions

① a) $x = 2 \cos \frac{2\pi}{3} = 2(-\frac{1}{2}) = -1$
 $y = 2 \sin \frac{2\pi}{3} = 2(\frac{\sqrt{3}}{2}) = \sqrt{3}$

b) $r^2 = (\sqrt{3})^2 + 1^2 = 4 \quad r = \pm 2$
 $\tan \theta = \frac{1}{\sqrt{3}}$

$(2, \frac{\pi}{6})$ or $(-2, \frac{7\pi}{6})$

② $x^2 - y^2 = 1$
 $(r \cos \theta)^2 - (r \sin \theta)^2 = 1$
 $r^2(\cos^2 \theta - \sin^2 \theta) = 1$
 $r^2 \cos 2\theta = 1$
 $r^2 = \frac{1}{\cos 2\theta} = \sec 2\theta$



⑤ $2 \cdot \frac{1}{2} \int_0^{\pi/4} (\cos 2\theta)^2 d\theta$
 $= \int_0^{\pi/4} \cos^2 2\theta d\theta$
 $= \int_0^{\pi/4} \frac{1 + \cos 4\theta}{2} d\theta$
 $= \frac{1}{2} \left[\theta + \frac{\sin 4\theta}{4} \right] \Big|_0^{\pi/4}$
 $= \frac{1}{2} \left(\frac{\pi}{4} \right) = \frac{\pi}{8}$

⑥ $\frac{1}{2} \int_0^{2\pi} [2(1 + \sin \theta)]^2 d\theta =$
 $2 \int_0^{2\pi} [1 + 2\sin \theta + \sin^2 \theta] d\theta =$
 $2 \int_0^{2\pi} \left[1 + 2\sin \theta + \frac{1 - \cos 2\theta}{2} \right] d\theta$
 $= 2 \int_0^{2\pi} \left[\frac{3}{2} + 2\sin \theta - \frac{\cos 2\theta}{2} \right] d\theta$
 $= 2 \left[\frac{3}{2} \theta - 2\cos \theta - \frac{\sin 2\theta}{4} \right] \Big|_0^{2\pi}$
 $= 6\pi$

Introduction to Polar Coordinates

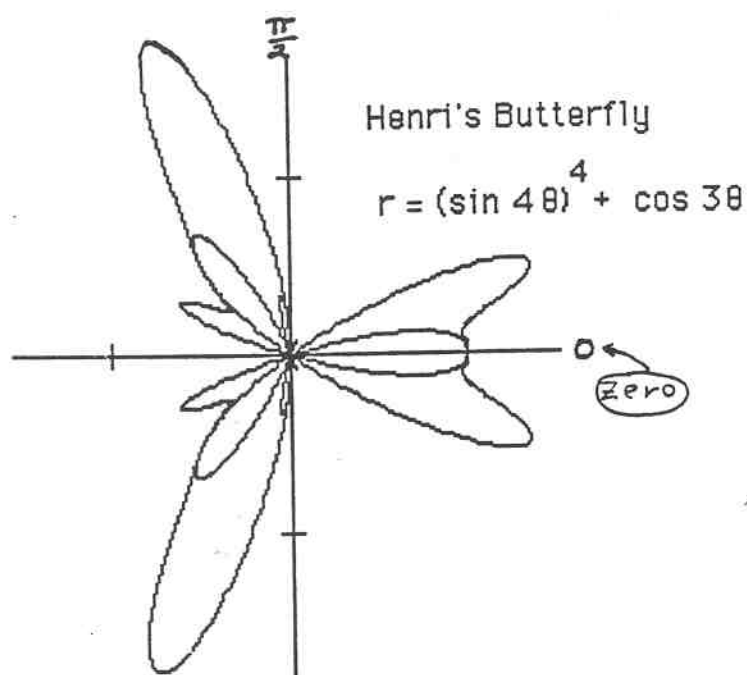


Figure 1

The polar coordinate graph was discovered by Henri Berger, a student in one of the author's (David Cohen) mathematics classes at UCLA, Spring 1988.

Polar Graphing Grid

