MAC 2311 EXAM I MR. NADEL SPRING 2012

(10)① a) What's the domain of

$$f(x)=\frac{2}{1-\sin x}\ ?$$

b) Write $y=f(x)=|x|+|x-1|$ in piecewise form <u>without</u> using absolute values, <u>Simplify</u>.
<u>Hint</u>: Use 3 cases.

(15)② a) Let $f(x)=\sqrt{x}$, $g(x)=x^2+1$.
Find and <u>simplify</u> $(g\circ f)(x)$ <u>and</u> its domain.

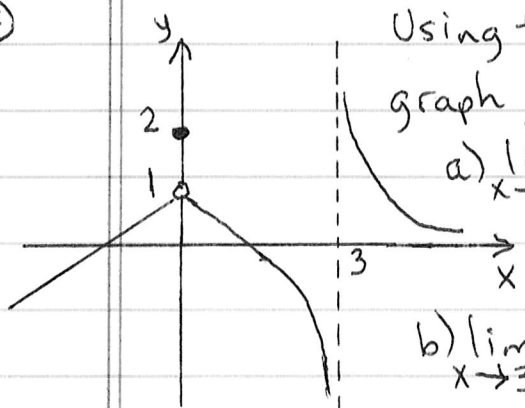
b) Sketch $y=\frac{1}{2}(x-2)^2$.
Include table of values <u>and</u> intercepts.

c) Express $h(x)=\cos^2 x$ as the composition of 2 functions f and g with $h=f\circ g$.
<u>Don't</u> let $f(x)=x$ or $g(x)=x$.

(10)③ a) Find the <u>exact</u> value of $\sin\left[2\cos^{-1}\left(\frac{1}{3}\right)\right]$ using $\sin 2x=2\sin x\cos x$.

b) If $y=f(x)=-\sqrt{2-x}$, find a <u>formula</u> for $f^{-1}(x)$. Be sure to <u>also</u> state the <u>domain</u> of f^{-1}.

(10)④



Using the graph, find

a) $\lim_{x\to 0} f(x)$

b) $\lim_{x\to 3^+} f(x)$

(15)⑤ a) $\lim_{x\to 0}\dfrac{x^2-2x}{x^2+x}$

b) $\lim_{x\to 3}\dfrac{x^3-27}{x-3}$

c) $\lim_{x\to 2^-}\dfrac{3}{x-2}$

(10)⑥ a) $\lim_{x\to-\infty}\dfrac{x^2-2x}{x^2+x}$

b) $\lim_{x\to\infty}\dfrac{2x^2-3}{5x^3+x+4}$

(15)⑦ a) Find a value of k that will make the function continuous everywhere: $f(x)=\begin{cases}3x+1 & x\le 2\\ kx^2 & x>2\end{cases}$

b) Where is f <u>not</u> continuous:

$$f(x)=\frac{x-3}{|x-3|}\ ?$$

<u>Also</u> state if essential or removable.

c) Approximate to within .05 the solution of $x^3-3x+1=f(x)=0$ that's between 1 and 2.

(15)⑧ a) $\lim_{x\to 0}\dfrac{\tan 4x}{x}$ <u>Don't</u> use L'Hôpital's Rule. Show work.

b) $\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)$

c) Where is $y=\tan^{-1}x$ continuous?

MAC 2311 EXAM I KEY (SP '12)

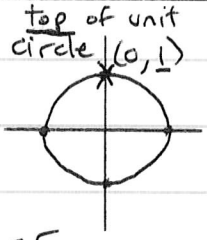
① a) $1-\sin x \neq 0$

$\sin x \neq 1$

$x \neq \frac{\pi}{2} + 2k\pi$

where k is any integer.

(positive, negative, or 0)



b) $x<0 \quad -x-(x-1) = -2x+1$

$0 \leq x \leq 1 \quad x-(x-1) = 1$

$x>1 \quad x+x-1 = 2x-1$

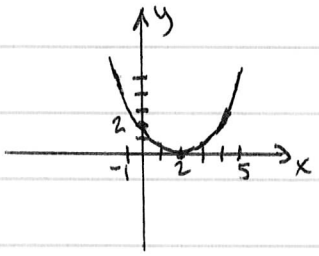
② a) $(g \circ f)(x) = g(\sqrt{x})$

$= (\sqrt{x})^2 + 1 = x+1$

$x \geq 0$ (so $\sqrt{x}$ is defined)

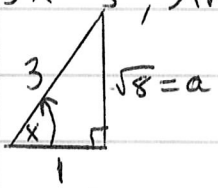
b)

x	y
-1	4.5
0	2
1	.5
2	0
3	.5
4	2
5	4.5



c) $f(x) = x^2$, $g(x) = \cos x$

③ a) $\cos x = \frac{1}{3}$, $\sin 2x = 2\sin x \cos x$



$a^2 + 1^2 = 3^2$

$a^2 = 9-1 = 8, \; a = \sqrt{8}$

$= 2\left(\frac{\sqrt{8}}{3}\right)\left(\frac{1}{3}\right)$

$= \frac{2\sqrt{8}}{9}$ or $\frac{4\sqrt{2}}{9}$

b) $y = -\sqrt{2-x}$. Note $y \leq 0$.

$x = -\sqrt{2-y}$ (switch x, y)

$x^2 = 2-y$

$y = 2-x^2$ Now, $x \leq 0$

$= f^{-1}(x)$.

④ a) 1

b) ∞

⑤ a) $\lim_{x \to 0} \frac{x(x-2)}{x(x+1)} = \frac{-2}{1} = -2$

b) $\lim_{x \to 3} \frac{(x-3)(x^2+3x+9)}{x-3}$

$= 3^2 + 3(3) + 9 = 27$

c) $-\infty$

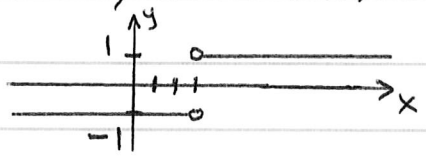
⑥ a) $\frac{1x^2}{1x^2} = 1$

b) 0 (degree den. > deg. num.)

⑦ a) $3(2)+1 = K(2)^2$

$7 = 4K \Rightarrow K = \frac{7}{4}$

b) $x = 3$, essential, (jump on graph)



c) $f(1) = -1 \quad f(2) = 3$

$f(1.5) = -.125$

$f(1.6) = .296$

sign change

Take $x = 1.55$

⑧ a) $\lim_{x \to 0} \frac{\sin 4x}{x(\cos 4x)}$

$= 4 \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot \frac{1}{\cos 4x}$

$= 4(1)(1) = 4$

b) $\cos 0 = 1$

c) everywhere (see graph)

or $(-\infty, \infty)$