

Ex. 5 (logarithmic)

Sketch $y = f(x) = \frac{\ln x}{x}$

Defined if $x > 0$.No symmetry. $\frac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \Rightarrow x = e^0 = 1$
(x intercept)

No y intercept.

$$\lim_{x \rightarrow 0^+} \frac{\ln x}{x} = -\infty \text{ since } \lim_{x \rightarrow 0^+} \ln x = -\infty$$

(not indeterminate)

There is a VA at $x = 0$

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0 \text{ by L.R. } \left(\lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0 \right)$$

$$f'(x) = \frac{x \left(\frac{1}{x}\right) - \ln x(1)}{x^2} = \frac{1 - \ln x}{x^2} \quad \begin{array}{l} 1 - \ln x = 0 \Rightarrow \\ \ln x = 1 \Rightarrow x = e \end{array}$$

 $\begin{array}{c} + \quad - \\ | \quad | \\ 0 \text{ inc. } e \text{ dec.} \end{array}$

$$f(e) = \frac{\ln e}{e} \approx 0.37$$

$$f''(x) = \frac{x^2 \left(-\frac{1}{x}\right) - (1 - \ln x)(2x)}{x^4} = \frac{2x \ln x - 3x}{x^4}$$
$$= \frac{2 \ln x - 3}{x^3}$$

Since $x > 0$, we need only look at $2 \ln x - 3$ $\begin{array}{c} | \quad | \\ 0 \text{ c.d. } e^{3/2} \text{ c.u.} \end{array}$

$$f(e^{3/2}) = \frac{3}{2e^{3/2}} \approx 0.33$$

approx.
4.48