Differentials Problems

① The side of a square is measured to be 10 ft., with a possible error of ± 0.1 ft.
a) Use differentials to estimate the error in the calculated area.
b) Estimate the percentage errors in the side and the area.
Answers: a) $\pm 2\ ft^2$ b) side $\pm 1\%$, area $\pm 2\%$

② The side of a cube is measured with a possible percentage error of $\pm 2\%$. Use differentials to estimate the percentage error in the volume.
Ans. $\pm 6\%$.

③ The area of a circle is to be computed from a measured value of its radius. Estimate the maximum permissible percentage error in the measurement if the percentage error in the area must be kept within $\pm 1\%$.
Ans. $\pm 0.5\%$.

(Solutions on next page)

Solutions

① a) $A = x^2$

$dA = 2x\,dx$

$= 2(10\text{ ft})(\pm 0.1\text{ ft})$

$= \pm 2\text{ ft}^2$

b) side: $\frac{\pm .1\text{ ft}}{10\text{ ft}} = \pm 1\% = \frac{dx}{x}$

$\frac{dA}{A} = \frac{2x\,dx}{x^2} = 2\left(\frac{dx}{x}\right) = 2(\pm 1\%)$

$= \pm 2\%$

② $V = x^3$

$\frac{dV}{V} = \frac{3x^2\,dx}{x^3} = 3\left(\frac{dx}{x}\right)$

$= 3(\pm 2\%) = \pm 6\%$

③ $A = \pi r^2$

$\frac{dA}{A} = \frac{2\pi r\,dr}{\pi r^2} = 2\left(\frac{dr}{r}\right)$

$\pm 1\% = 2\left(\frac{dr}{r}\right)$

$\pm 0.5\% = \frac{dr}{r}$