

Homework Assignments

p10 1-59 odd, 61-67 odd, 73, 75

p24 1-37 odd, 39-47 odd

p39 1-35 odd, 37-45 odd

p56 1-15 odd, 29, 33, 35, 37, 39, 43, 45a, 49

p73 1-37 odd

p86 1-41 odd, 53

p112 1-31 odd

p125 1-49 odd, 51-57 odd, 61, 63

p138 1-57 odd

p151 1-67 odd

p163 1-23 odd

Review Exam I

p202 1-49 odd

p220 1-35 odd

p234 1-19 odd 23

p254 1-15 odd, 17a, 19a 23, 25

p270 1-11 odd, 17, 23, 25, 27

p303 1-49 odd

p320 1-45 odd, 53, 55, 57

p335 1-33 odd, 39, 41, 47, 57-63 odd

p349 21-25 odd, 29, 31, 33, 39

Review Exam II

p 381 1-41 odd, 43, 45, 49

p 394 1-35 odd, 47-53 odd

p 410 1-29 odd

p 427 1-17 odd

p 442 1-9 odd, 21, 23

p 568 1-21 odd

p 583 1-33 odd, 45, 47

p 596 1-10, 11, 15, 17* (Double quiz and final exam)
only

Review Exam III

Answers to even numbered problems p.

2. $f(0,0)$ is a saddle point
4. $f(-2,-4)$ is a relative minimum
6. $f(2,2)$ is a relative minimum
8. $f(1,-1)$ is a saddle point
10. $f(1,-3)$ is a relative minimum
10. $f(-2,2)$ is a saddle point
10. $f(-2,-2)$ is a relative maximum

MAC 2233
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Review Exam I

1. A manufacturer buys \$20,000 worth of machinery that depreciates linearly so that its trade-in value after 10 years will be \$1000.

(a) Express the value of the machinery as a function of its age and draw the graph.

(b) Compute the value of the machinery after 4 years.

2. Find the equilibrium price and the corresponding number of units supplied and demanded if the supply function for a certain commodity is

$S(p) = p^2 + 3p - 70$ and the demand function is $D(p) = 410 - p$.

3. Under the provisions of a proposed property tax bill, a homeowner will pay \$100 plus 8 percent of the assessed value of the house. Under the provisions of a competing bill, the homeowner will pay \$1900 plus 2 percent of the assessed value. If only financial considerations are taken into account, how should a homeowner decide which bill to support?

4. A manufacturer has been selling computers at \$1500 and that price consumers have been buying 2000 a month. The manufacturer wishes to lower the price to stimulate sales, and estimates that for every \$100 decrease in price, 25 more computers will be sold each month. the cost of the computer is \$900. Express manufacturer's monthly profit as a function of price, x, or as a function of y, the number of \$100 decreases.

5. Farmers can get \$2 per bushel for their potatoes on July first, and after that, the price drops by 2 cents per bushel per day. On July first, a farmer has 80 bushels of potatoes in the field and estimates that the crop is increasing at the rate of 1 bushel per day. Express the farmer's revenue from the sale of the potatoes as a function of the time at which the crop is harvested.

6.a) A closed box with square base has volume of 1500 cubic inches.

Express its surface area as a function of the length of its base.

b) A closed box with a square base is to have a volume of 250 cubic meters. The material for the top and bottom of the box costs \$2 per square meter, and

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the material for the sides costs \$1 per square meter. Express the construction cost of the box as a function of the length of its base.

7. a) A cylindrical can is to hold 4π cubic inches of frozen orange juice. The cost per square inch of the metal top and bottom is \$4 per square inch. The cost of the cardboard side is \$2 per square inch. Express the cost of constructing the can as a function of its radius.

b) A closed cylindrical can has surface area 120π square inches. Express the volume of the can as a function of its radius.

8. An environmental study of a certain suburban community suggests that t years from now, the average level of carbon monoxide in the air will be $Q(t) = .05t^2 + .1t + 3.4$ parts per million.

a) At what rate will the carbon monoxide level be changing with respect to time 1 year from now?

b) At what percentage rate will the carbon monoxide level be changing with respect to time 1 year from now?

9. When a certain commodity is sold for p dollars per unit, consumers will buy $D(p) = \frac{40,000}{p}$ units per month. It is estimated that t months from now,

the price of the commodity will be $p(t) = 0.4t^{3/2} + 6.8$ dollars per unit. At what rate will the monthly demand be changing with respect to time 4 months from now?

10. At a certain factory the daily output is $Q(L) = 60,000 L^{3/4}$ units, where L is the size of the labor force measured in worker hours. Currently 1000 worker hours of labor are used each day. Estimate the effect on output if the labor force is cut to 940 worker hours.

11. Find the equation of the line that is tangent to the graph of the given function at the specified point:

a) $f(x) = \frac{x+1}{x-1}$ (0, -1)

b) $f(x) = (x^3 - 2x^2 + 3x - 1)(x^5 - 4x^2 + 2)$ $x = 0$

12. Find $f'(x)$ where

a) $f(x) = \sqrt{\frac{2x+5}{x^2-4}}$

b) $f(x) = (3x+1)^7 \sqrt{5x^2-2}$

c) $f(x) = (\sqrt{x^3+8x^2})(3x^4-2)^5$

d) $f(x) = \frac{(\sqrt{x}-4)^3}{x^2-5x^3}$

13. Find the second derivative of $f(x) = 2x(x+4)^3$ 5

14. Find the following limits

a) $\lim_{x \rightarrow 2} \frac{x}{(x-2)^2}$ b) $\lim_{x \rightarrow 16} \frac{x-16}{\sqrt{x}-4}$ c) $\lim_{x \rightarrow 1} \frac{x^2+4x-5}{x^2-1}$

d) $\lim_{x \rightarrow +\infty} \frac{3x^3+2x^2-5x+1}{5x^3-2}$ e) $\lim_{x \rightarrow +\infty} \frac{-2x^2+4x-3}{x^3+2x^2-2x-1}$

15. Find a value of A so that $f(x)$ will be continuous

$$f(x) = \begin{cases} 1-3x & \text{if } x \leq 4 \\ Ax^2+2x-3 & \text{if } x > 4 \end{cases}$$

16. Find points of discontinuity (if any) for

a) $f(x) = \frac{2x+1}{3x-6}$

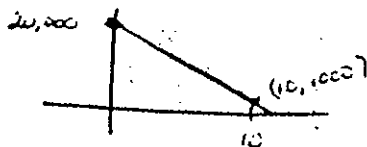
b) $f(x) = \begin{cases} 0 & \text{if } x < 1 \\ x^2+1 & \text{if } x \geq 1 \end{cases}$

Solutions Review Exam I

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1 a) $V(t) = -1900t + 20,000$ where $V(t)$ = value of machine in t years

b) $V(4) = 12,400$



2 $p = 20$ $D(20) = 390$

3. Let x = assessed value of house

$T_1(x) = 100 + .08x$ $T_2(x) = 1900 + .02x$

If $x > 30,000$ Choose second plan; If $x < 30,000$ Choose first plan

4. $P(x) = (x-900)(2000 + 25(\frac{1500-x}{100})) = (x-900)(2125 - \frac{1}{4}x)$

$P(y) = (1500 - 100y - 900)(2000 + 25y) = (600 - 100y)(2000 + 25y)$

5 $R(t) = (2 - .02t)(80 + t)$ where t = # days after July 1

6 a) $S(x) = 2x^2 + \frac{6000}{x}$ b) $S(x) = 4x^2 + \frac{1000}{x}$

7. a) $C(r) = 10\pi r^2 + \frac{16\pi}{r}$ (in cents)

b) $V(r) = 60\pi r - \pi r^3$

8 a) $Q'(1) = .2$ parts/million/yr b) 5.63% / year

9. $D'(p) = \frac{-40,000}{p^2}$ $p'(t) = .16t^2$ $p(4) = 10$ $\frac{dD}{dt} \Big|_{p=4} = (D'(10)) \times p'(4) = -480$ units/mc

10. $Q'(L) = 20,000 L^{-2/3}$ $L = 1000$, $\Delta L = -60$

$\Delta Q \approx Q'(L) \Delta L = \frac{20,000}{(\sqrt[3]{1000})^2} (-60) = -12,000$ units

11 a) $f'(x) = \frac{-2}{(x-1)^2}$, $f'(0) = -2$; $y+1 = -2(x-0)$

b) $f'(0) = 6$, $y = 6x - 2$

12 a) $f'(x) = \frac{1}{2} \sqrt{\frac{x^2-4}{2x-5}} \left(\frac{-2x^2-10x-5}{(x^2-4)^2} \right)$

b) $f'(x) = \frac{5x(3x+1)^4 + 12(5x^2-2)(3x+1)^3}{\sqrt{5x^2-2}}$

c) $f'(x) = \frac{120x^3(x^3+8x^2)(3x^4-2)^4 + (3x^4-2)^5(3x^2+16x)}{2\sqrt{x^3+8x^2}}$

d) $f'(x) = \frac{(x^2-5x^3)(3\sqrt{x-4})^2 \frac{1}{2\sqrt{x}} - (\sqrt{x-4})^3(2x-15x^2)}{(x^2-5x^3)^2}$

13 a) $f'(x) = 8(x+4)^2(x+1)$

$f''(x) = 24(x+4)(x+2)$

14 a) $+\infty$ b) 8 c) 3 d) $\frac{3}{j}$ e) 0

15 $A = -1$

16 a) $X = 2$

b) $X = 1$

1. Find critical points, intervals of increase and decrease, relative extrema, second order critical points, intervals of concave up and down, inflection points and sketch graph for

a) $f(x) = x^4 - 2x^2 + 3$

b) $f(x) = x^5 - 5x^4 + 100$

2. Find the absolute maximum and absolute minimum for the function on the interval:

a) $f(x) = (x+1)^{2/3}$ on $-2 \leq x \leq 1$

b) $f(x) = \frac{x^2}{x+1}$ on $-\frac{1}{4} \leq x \leq 1$

3. A computer manufacturer can sell 1500 personal computers per month at a price of \$3000 each. The manager estimates that for each \$200 price reduction he will sell 300 more each month. Find the price of the computer that will maximize revenues.

4. A baseball card store can obtain rookie cards at a cost of \$5 per card. The store has been offering the cards at \$10 apiece, and at this price has been selling 50 cards per month. The store is planning to lower the price to stimulate sales and estimates that for each 50-cent reduction in the price, 5 more cards will be sold each month. At what price should the cards be sold to maximize total monthly profit?

5. A company wishes to design an open-top box with a square base whose volume is 64 cubic feet. The materials for the sides cost \$2 per square foot and the material for the base costs \$4 per square foot. Find the dimensions of the box that will minimize cost.

6. A chemical manufacturer wants to produce a closed cylindrical can with a volume of 1024 cubic centimeters. What dimensions will minimize the surface area of the can?

7. Find $\frac{dy}{dx}$ where

a) $y = \ln(\sqrt{x} + e^x)$

b) $y = \frac{e^{\sqrt{x-2}}}{\ln x}$

8. Use logarithmic differentiation to find $\frac{dy}{dx}$

where $y = \frac{(x^2+1)^{\frac{1}{3}} \sqrt{x^2-5}}{e^{x^2}}$

9. What is the present value of \$10,000 over a period of 5 years if interest is compounded continuously at an annual rate of 7%?

10. One day on a college campus when there were 5000 people in attendance, a particular student heard that a certain controversial speaker was going to make an unscheduled appearance. This information was told to friends who in turn, related it to others. After t minutes elapsed, $f(t)$ people heard the rumor where $f(t) = \frac{5000}{1 + 4999e^{-5t}}$

a) How many people heard the rumor after 10 minutes?

b) How many people will ultimately hear the rumor.

c) Sketch the graph of $y = f(t)$

11. The increase of a population in a certain city is growing exponentially. If the population doubles in 60 years and was 60,000 in 1980, when will the population be 150,000.

12. A strontium isotope has a half life of 90 years. How long will it take 50 grams of strontium to decay to 5 grams.

13. The total cost of producing x units of a commodity is $C(x) = 40e^{x/4}$. Find the minimum average cost per unit ($A(x) = \frac{C(x)}{x}$)

14. Consumer demand for a commodity is estimated to be $D(p) = 25,000e^{-0.02p}$ units per month where p is the selling price in dollars. Find the selling price that will maximize expenditure, $E(p) = pD(p)$.

15. Find the vertical and horizontal asymptotes for

a) $g(x) = \frac{5x^2}{x^2 - 3x - 4}$

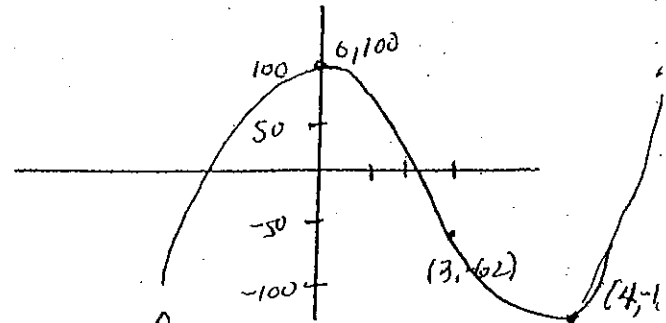
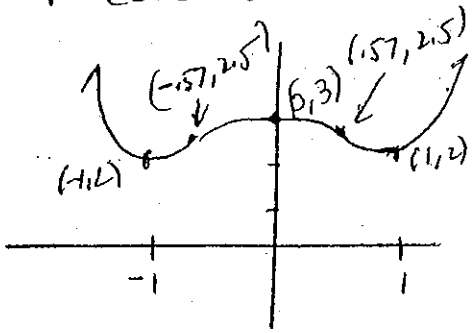
b) $h(x) = \frac{-2x}{x^2 - 4}$

c) $f(x) = \frac{x^3 + 2x^2 + 1}{x - 1}$

16. Compute the elasticity of demand for $D(p) = -1.5p + 25$ and determine whether the demand is elastic, inelastic or of unit elasticity for $p = 12$

Solutions

1 a) C.P. $x=0, 1, -1$; f decreases $x < -1$ or $0 < x < 1$; f increases $-1 < x < 0$ or $x > 1$; Rel min $(-1, 2), (1, 2)$; Rel max $(0, 3)$; S.O.C.P $x = \pm \sqrt{\frac{1}{3}} \approx \pm .57$; f concave up $x < -.57$ or $x > .57$; f concave down $-.57 < x < .57$; Inflection pts $(-.57, 2.5), (.57, 2.5)$



1 b) C.P. $x=0, 4$; f decreases $0 < x < 4$; f increases $x < 0$ or $x > 4$; Relative min $(4, -156)$; Rel max $(0, 100)$; S.O.C.P. $x=3$; f concave down $x < 3$, f concave up $x > 3$; Inflection pt $(3, -62)$

Note: S.O.C.P = second order critical points

2 a) C.P. $x = -1$

x	$f(x)$
-2	1
-1	0 ← Min
1	1.59 ← Max

b) C.P. $x=0$ ($x=-2$ outside interval)

x	$f(x)$
$-\frac{1}{4}$	$\frac{1}{12}$
0	0 ← Min
1	$\frac{1}{2}$ ← Max

3. Let $x =$ price $R(x) =$ revenue; $R(x) = x \left(1500 + 300 \left(\frac{3000-x}{200} \right) \right)$
 $x = \$2000$ maximizes revenue

4. Let $x =$ price $P(x) =$ profit $P(x) = (x-5) \left(50 + \left(\frac{10-x}{1.5} \right) 5 \right)$
 $x = \$10$ will maximize profit

5. $x =$ side of base $y =$ height $C(x) =$ cost $C(x) = \frac{512}{x} + 4x^2$
 $x = y = 4$ will minimize cost

6. $r =$ radius $h =$ height $S(r) =$ surface area $S(r) = \frac{2048}{r} + 2\pi r^2$
 $r = \frac{8}{\sqrt[3]{\pi}}$ $h = \frac{16}{\sqrt[3]{\pi}}$ will minimize surface area

7 a) $\frac{dy}{dx} = \frac{\frac{1}{2\sqrt{x}} + e^x}{\sqrt{x} + e^x}$

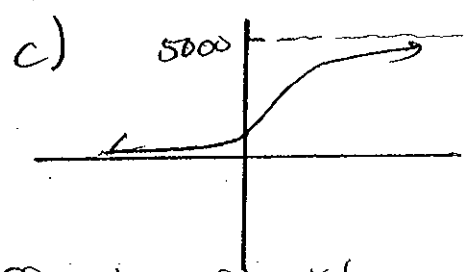
b) $\frac{dy}{dx} = \frac{\ln x \left(\frac{1}{2\sqrt{x-2}} \cdot e^{\sqrt{x-2}} \right) - e^{\sqrt{x-2}} \cdot \frac{1}{x}}{(\ln x)^2}$

8. $\ln y = \frac{1}{3} \ln(x^2+1) + \frac{1}{2} \ln(x^3-5) - x^2$

$\frac{d \ln y}{dx} = \left[\frac{2x}{3(x^2+1)} + \frac{3x^2}{2(x^3-5)} - 2x \right] \frac{(x^2+1)^{\frac{1}{3}} \sqrt{x^3-5}}{e^{x^2}}$

9. $P = 10,000 e^{-.07(5)} = \7046.88

- 10 a) 144
- b) 5000



11. $Q(t) = Q_0 e^{kt}$ $k = .01155$ $t \approx 79$ years from 1980 (2059)

12. $Q(t) = Q_0 e^{-kt}$ $k = .0077$ $t \approx 299$ years

13. $A'(x) = \frac{40e^{\frac{x}{4}} (\frac{x}{4} - 1)}{x^2} = 0$ when $x = \$4$

$A'(x)$ $\frac{-}{4} \frac{+}{}$ so $A(4)$ is minimum

14. $E'(p) = 25,000 e^{-.02p} (-.02p + 1) = 0$ when $p = \$50$

$E'(p)$ $\frac{+}{50} \frac{-}{}$ so $E(50)$ is maximum

15 a) Vertical Asymptotes $x=4, x=-1$
Horizontal Asymptote $y=5$

b) Vertical $x=2, x=-2$
Horizontal $y=0$

c) Vertical $x=1$
No horizontal asymptote

1. Evaluate the following integrals

a) $\int (\pi + \frac{5}{x} - \frac{3}{2x^4} + e^{3x} + \sqrt{x}) dx$

b) $\int \frac{(3x^3 + 6x) dx}{\sqrt{x^4 + 4x^2 + 6}}$

c) $\int (3x^2 - 1) e^{(x^3 - x)} dx$

d) $\int \frac{(x^2 - 1) dx}{3x^3 - 9x}$

e) $\int_1^2 \frac{x^4 dx}{(x^5 + 1)^3}$

f) $\int_2^{e^2} \frac{dx}{x(\ln x)}$

2. Oil will be pumped from a producing field at a rate given by $\frac{120t}{t^2 + 1} + 3$ thousands of barrels per year, t years after pumping begins. How many barrels will be produced during the first five years?
3. A manufacturer estimates marginal revenue to be $100q^{-\frac{1}{2}}$ dollars per unit when level of production is q units, and marginal cost is $.4q$ dollars per unit. Suppose profit is \$520 for 16 units. What is the profit when level of production is 25 units?
4. Sketch the region and calculate the area of the region bounded by $y = 4 - x^2$ and $y = 3x$

5. For the consumer demand function $p = D(q) = 400 - \frac{q}{20}$ 14
- Find the demand q , when $p = \$150$.
 - Find the "willingness to buy" for that q .
 - Find the consumer's surplus for that q .

6. For $z = \frac{x^3y+5}{xy^3}$ a) Find $\frac{dz}{dx}$ b) $\frac{dz}{dy}$

7. For $f(x,y) = x^2e^y + y \ln x$, find all four second order partial derivatives.

Review for Double quiz and FINAL (only)

8. Find the critical points of the given function and classify each as a relative maximum, relative minimum, or a saddle point.

a) $f(x,y) = 2x^2 - 3y^2$

b) $f(x,y) = xy + \frac{8}{x} + \frac{8}{y}$

c) $f(x,y) = -x^4 - 32x + y^3 - 12y + 7$

9. A manufacturer estimates that annual output at a factor is $Q(K,L) = 30K^{.3}L^{.7}$ units where K is capital expenditure in thousands and L is the size of the labor force in worker hours. Currently, expenditure is \$630,000 and labor is 830 worker hours. Use marginal analysis to estimate the increase in output if capital expenditure is increased by \$1000 (worker hours remain unchanged)

Solutions - Review Exam III

1 a) $\pi x + 5 \ln|x| + \frac{1}{2x^3} + \frac{1}{3}e^{3x} + \frac{3}{7}\sqrt[3]{x^4} + C$

b) $\frac{3}{2}\sqrt{x^4 + 4x^2 + 6} + C$

c) $e^{x^2-x} + C$

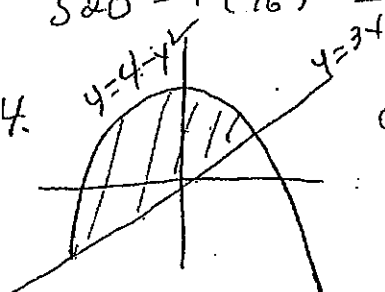
d) $\frac{1}{7} \ln|3x^3 - 9x| + C$

e) $\frac{217}{8712} \approx 0.0249$

f) $\ln 2 - \ln(\ln 2) \approx 1.05966$

2. $\int_0^5 \left(\frac{120t}{t^2+1} + 3 \right) dt = 60 \ln|(t^2+1)| + 3t \Big|_0^5 = 60 \ln(26) + 15 - 0 \approx 210.486$ thousand barrels

3. $P(q) = \int (100q^{-1/2} - .4q) dq = 200\sqrt{q} - .2q^2 + C$
 $520 = P(16) \rightarrow C = -228.8$ so $P(25) = \$646.20$



$4 - x^2 = 3x$
 $0 = x^2 + 3x - 4$
 $0 = (x+4)(x-1)$
 $x = -4, 1$

$\int_{-4}^1 (4 - x^2 - 3x) dx = 4x - \frac{x^3}{3} - \frac{3x^2}{2} \Big|_{-4}^1 = \frac{125}{6}$

5 a) $q = 5000$ b) $\$1,375,000$ c) $\$625,000$

6 $\frac{dz}{dx} = \frac{2x^3y^4 - 5y^3}{x^2y^6}$ $\frac{dz}{dy} = \frac{-2x^4y^3 - 15xy^2}{x^2y^6}$

7. $f_x = 2xe^y + \frac{y}{x}$; $f_{xx} = 2e^y - \frac{y}{x^2}$; $f_{xy} = 2xe^y + \frac{1}{x}$
 $f_y = x^2e^y + \ln x$; $f_{yx} = 2xe^y + \frac{1}{x}$; $f_{yy} = x^2e^y$

8. a) $(0,0)$ saddle point b) $(2,2)$ relative minimum
 (Note: $x=0$ not in domain)
 c) $(-2,2)$ - saddle point; $(-2,-2)$ relative maximum

9. $\Delta Q \approx Q_K(K,L) \Delta K$ $Q_K(K,L) = \frac{30(3)L^{.7}}{K^{.1}} (\Delta K)$ $\Delta K = 1$
 $Q_K = 9(830)^{.7} (1) = 10.9$ units (1 thousand)