Homework Problems
p. 12 1,3,7,9,15-23 odd, 29a, 31ab, 33 (express cost C, in terms of radius, r)
p.24 1a,b,3a,d, 9, 13, 17, 27-37 odd, 35,
p. 35 11, 29-35 odd
p. 48 1-19 odd, 31-41 odd
p. 61 1,5,9-27 odd
p. 77 1-9odd, 17-25odd
p.87 1-31 odd, 37, 43
p. 97 1-5odd, 9-39odd, 47, 51, 55-61odd
p.107 13=21, odd, 29, 31-35odd, 39b,c
p.109 49, 51, 53, 57-61 odd
p.118 1-5odd, 11-21 odd, 29, 31, 35, 47
p. 125 1-39odd, 51-55odd, 67a
Review Exam I

p. 140 3, 11-17 odd, 27
p. 152 7-33 odd, 47, 49 (Use Squeeze Theorem)
p. 161 1-23 odd 29-45 odd, 65-59 odd
p. 168 1-21 odd 29-33 odd
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p. 195 1-43 odd
p. 201 1-9 odd, 15-51 odd
p. 208 1, 3, 11-21 odd, 25-33 odd, 39
p. 217 1c, 23-33 odd, 51-65 odd
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p. 252 1-11 odd, 21-29 odd, 33-43 odd, 51, 53
p. 264 1,3,15, 33, 45
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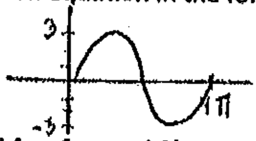
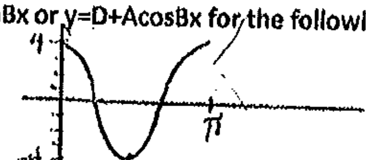
Review Exam III

Review Exam I

1. A manufacturer constructs open boxes from sheet s of cardboard that are 6 inches square by cutting small squares from the corners and folding up the sides. Determine a function of volume, V(x) where x is the length of the side to be cut.
2. A soup company wants to manufacture a can in the shape of a right circular cylinder that will cost 25 cents. The cost of the top and bottom is 2 cents/cm ; the cost of the bottom is 1 cent/cm . Express the volume of the can, V(r) as a function of the radius, r.
3. Find the domain of $G(x) = \sqrt{\dfrac{x^2-4}{x-4}}$.
4. Use the graph of $y = |x|$ to graph
   a) $Y = 3-|x+4|$
   b) $Y = \sqrt{x-3}+2$
5. Find an equation in the form y=D+AsinBx or y=D+AcosBx for the following two graphs:

a) 

b) 

6. Find f+g, f·g, and f/g and give the domain of these functions where $f(x)=\sqrt{x+1}$ and $g(x)=\sqrt{5-x}$
7. Find $f\circ g$ and its domain where $f(x)= -x^2$ and $g(x)= 1/\sqrt{x}$
8. Find $f^{-1}(x)$ where $f(x) = \sqrt[3]{4x+2}$
9. Complete the identity:
   a) $\sin(\cos^{-1} x)=$
   b) $\tan(\cos^{-1} x)=$
   c) $\sec(\tan^{-1} x)$
10. Solve for x:
    a) $\ln(4x)-3\ln(x^2)= \ln(2)$
    b) $3e^{-2x} = 5$ (approximate with a calculator)
11. Calculate the following limits:
    a) $\lim\limits_{x\to 3} \dfrac{x^2-x-6}{x-3}$
    b) $\lim\limits_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

c) $\lim\limits_{x\to 3^-} \dfrac{x}{x-3}$

d) $\lim\limits_{x\to 2^+} \dfrac{x}{x^2-4}$

e) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{3x^4+x}}{x^2-8}$

f) $\lim\limits_{x\to \infty} \dfrac{1-e^x}{1+e^x}$

g) $\lim\limits_{x\to \infty} (1+\dfrac{1}{x})^{-x}$

h) $\lim\limits_{x\to 0} \dfrac{\sin x^2}{x^2} + \dfrac{\sin 2x}{\sin 5x}$

i) $\lim\limits_{x\to 0} \dfrac{\sin(\pi+x)}{x}$

j) $\lim\limits_{x\to 0} \dfrac{\tan 7x}{\sin 3x}$

k) $\lim\limits_{h\to 0} \dfrac{1-\cos 3h}{\cos^2 5h-1}$

l) $\lim\limits_{x\to 3}$ where $f(x) = \begin{cases} x^2-1, & x<3 \\ (x-1)^3, & x\geq 3 \end{cases}$

12. Using the formal definition of $\lim\limits_{x\to a} f(x)=L$:

a) Prove $\lim\limits_{x\to -1} 2-3x=5$

b) Prove $\lim\limits_{x\to \infty} \dfrac{1}{x+2}=0$

c) Prove $\lim\limits_{x\to 1} \dfrac{1}{|x-1|}=+\infty$

13. Find Let $g(x) = \begin{cases} \dfrac{2x^2+3x+1}{x+1}, & x<-1 \\ \dfrac{|x|}{x}, & -1 \le x < 0 \\ 2x, & x \ge 0 \end{cases}$

Where is g(x) continuous?

14. Assign a value to k to make g(x) continuous (everywhere)

$$g(x) = \begin{cases} \dfrac{x+2}{x^3+2x^2+x+2}, & x \neq -2 \\ k, & x = -2 \end{cases}$$

15. Find values of x, if any, at which f is not continuous

$$f(x) = \frac{5}{x} + \frac{2x}{x+4}$$

Review Exam I - Solutions

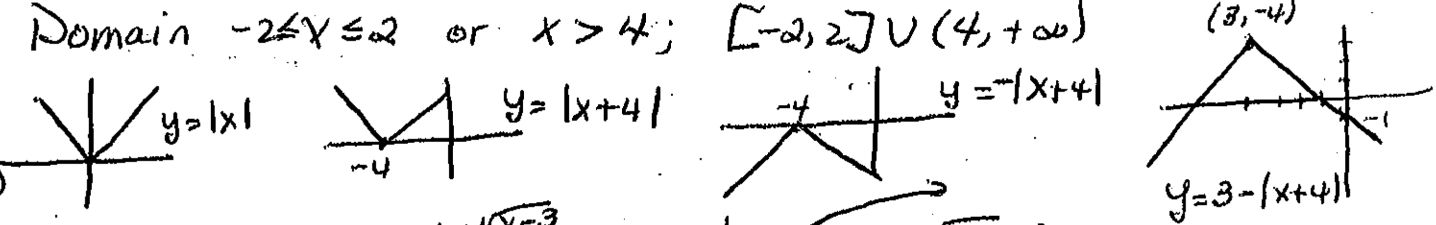
1. $V(x) = x(6-2x)^2$

2. $2(2\pi r^2) + 1(2\pi r h) = 25$; $\quad h = \dfrac{25-4\pi r^2}{2\pi r}$

$V(r) = \pi r^2 h = \pi r^2\left(\dfrac{25-4\pi r^2}{2\pi r}\right) = \dfrac{25}{2} r - 2\pi r^3$

3. Domain $-2 \le x \le 2$ or $x > 4$; $[-2, 2] \cup (4, +\infty)$

4. a)



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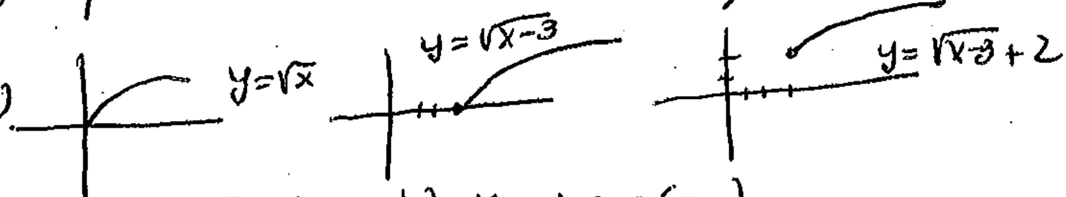


5a $y = 3\sin\left(\frac{1}{2}x\right)$ $\quad$ b) $y = 4\cos(2x)$

6. $(f+g)(x) = \sqrt{x+1} + \sqrt{5-x}$, $\quad f\cdot g(x) = \sqrt{-x^2+4x+5}$ $\quad$ Domain $[-1, 5]$

$(f/g)(x) = \sqrt{\dfrac{x+1}{5-x}}$ $\quad$ Domain $[-1, 5)$

7. $f\circ g(x) = f\left(\frac{1}{\sqrt{x}}\right) = -\left(\frac{1}{\sqrt{x}}\right)^2 = \frac{-1}{x}$ $\quad$ Domain $(0, +\infty)$

8. $f^{-1}(x) = \dfrac{x^5-2}{4}$

9 a) $\sqrt{1-x^2}$ $\quad$ b) $\dfrac{\sqrt{1-x^2}}{x}$ $\quad$ c) $\sqrt{1+x^2}$

10 a) $x = \sqrt[5]{2}$ $\quad$ b) $-.2554$

11 a) 5 $\quad$ b) 6 $\quad$ c) $-\infty$ $\quad$ d) $+\infty$ $\quad$ e) $\sqrt{3}$ $\quad$ f) -1 $\quad$ g) e^{-1} $\quad$ h) $\frac{7}{5}$

i) -1 $\quad$ j) $7/3$ $\quad$ k) $\frac{-9}{50}$ Hint: Multiply by $\frac{(1+\cos 3h)}{(1+\cos 3h)}$ $\quad$ l) 8

12 a) Given $\varepsilon > 0$, find $\delta > 0$

$|2-3x-5| < \varepsilon$ if $|x+1| < \delta$

$|-3-3x| < \varepsilon$ if $|x+1| < \delta$

$|-3|\,|x+1| < \varepsilon$ if $|x+1| < \delta$

$|x+1| < \frac{\varepsilon}{3}$ if $|x+1| < \delta = \frac{\varepsilon}{3}$

b) Given $\varepsilon > 0$ find $\delta > 0$,

$\left|\frac{1}{x+2} - 0\right| < \varepsilon$ if $x > N$

$|x+2| > \frac{1}{\varepsilon}$ if $x > N$

$|x| > \frac{1}{\varepsilon} - 2$ if $x > \frac{1}{\varepsilon} - 2 = N$

c) Given $M > 0$ find $\delta > 0$,

$\left|\frac{1}{x-1}\right| > M$ if $|x-1| < \delta$

$|x-1| < \frac{1}{M}$ if $|x-1| < \delta = \frac{1}{M}$

13. $g(x)$ is continuous except possibly at $x = -1$ and/or 0

$$\lim_{x \to -1^-} \frac{2x^2+3x+1}{x+1} = \lim_{x \to -1^-} \frac{(2x+1)(x+1)}{(x+1)} = -1$$

$$\lim_{x \to -1^+} \frac{|x|}{x} = \lim_{x \to -1^+} \frac{-x}{x} = -1 = f(-1)$$

So $g(x)$ is continuous at $x = -1$

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = -1 \qquad \lim_{x \to 0^+} 2x = 0 = f(0)$$

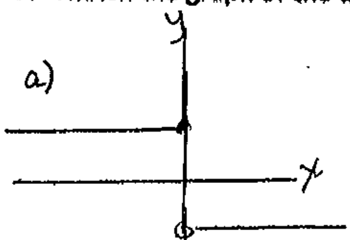
so $\lim_{x \to 0} g(x)$ does not exist. $g(x)$ is discontinuous at $x = 0$

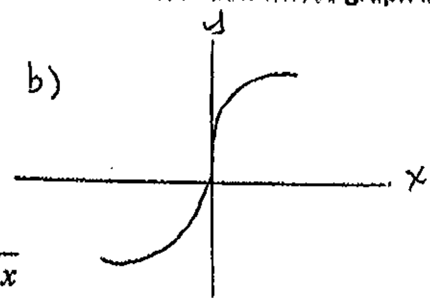
14. $$\lim_{x \to -2} g(x) = \lim_{x \to -2} \frac{(x+2)}{(x+2)(x^2+1)} = \lim_{x \to -2} \frac{1}{4+1} = \frac{1}{5}$$

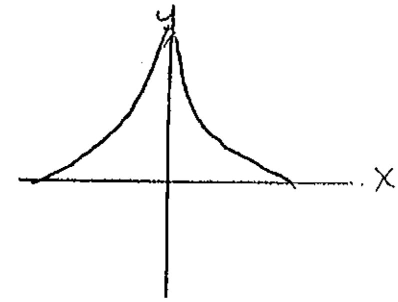
15. f is not continuous at $x = 0$, $x = -4$

Review Exam II

1. Use the definition of the derivative (only) to find f′(x) where $f(x) = -2/x^2$.
2. An automobile is driven down a straight highway such that after $0 \le t \le 12$ seconds it is $s = 4.5t^2$ feet from its initial position.
   a) Find the average velocity of the car over the interval [0,12].
   b) Find the instantaneous velocity of the car at time t = 6.
3. Sketch the graph of the derivative of the function whose graph is shown:







4. Find $\frac{dy}{dx}$ where $y = \frac{7}{x^6} - 5\sqrt{x}$

5. Let $f(x) = \begin{cases} x^3 + \frac{1}{16}, & x < \frac{1}{2} \\ \frac{3}{4}x^2, & x \ge \frac{1}{2} \end{cases}$

   Determine whether f is differentiable at $x = \frac{1}{2}$. If so, find the value of the derivative there.

6. Find $\left.\frac{dy}{dx}\right|_{x=1}$ for $y = (2x^7 - x^2)\left(\frac{x-1}{x+1}\right)$
7. Find f′(x)
   a) $f(x) = \frac{\sin x}{x^2 + \sin x}$
   b) $f(x) = \cos x - x \csc x$
   c) $f(x) = \sec^2 x - \tan^2 x$
8. Find f′(x)
   a) $f(x) = (3x^2 + 2x - 1)^6$
   b) $f(x) = \sqrt{x^3 - 2x + 5}$
   c) $f(x) = \tan^4(x^3)$

9. Find $\frac{dy}{dx}$ where y = $\frac{\sin x}{\sec(3x+1)}$

10. Find an equation of the tangent line to y = $3\cot^4 x$ at x = $\pi/4$.

11. Find $\frac{dy}{dx}$ by differentiating implicitly:

a) $\cos(xy^3) = y$

b) $\frac{xy^3}{1+\sec y} = 1 + y^4$

12. Find $\frac{d^2y}{dx^2}$ for $x^3 + y^3 = 1$

13. Find $\frac{dy}{dx}$

a) y= $\ln\left|x^3 - 7x^2 - 3\right|$

b) y= $\sin^2(\ln x)$

14. Find $\frac{dy}{dx}$ using logarithmic differentiation.

$$Y = \frac{\sin x \cos x \tan^3 x}{\sqrt{x}}$$

15. Determine whether f(x)= x^5+8x^3+2x+1 is one-to-one by examining the sign of $f'(x)$

16. Find the derivative of $f^{-1}(x)$ for f(x) = $\frac{1}{x^2}$, x>0 (using $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$)

17. Find $\frac{dy}{dx}$.

a) $y = \ln(\cos e^x)$

b) $y = (x^2+3)^{\ln x}$

c) $y = \cos^{-1}(\frac{x+1}{2})$

d) $y = \sec^{-1}(x^5)$

e) $y = \frac{1}{\tan^{-1} x}$

f) $y = x^2(\sin^{-1} x)^3$

18. A spherical balloon is to be deflated so that its radius decreases at a constant rate of 15 cm/min. At what rate must air be removed when the radius is 9cm?
19. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5 ft/s, how fast will the top of the ladder be moving down the wall when it is 8 ft above the ground?
20. Grain pouring from a chute at the rate of $8\text{ft}^3/\text{min}$ forms a conical pile whose height is always twice its radius. How fast is the height of the pile increasing at the instant when the pile is 6 ft high?
21. Use an appropriate local linear approximation to estimate $(1.97)^3$.
22. The volume of a sphere is to be computed from a measured value of its radius. Estimate the maximum permissible percentage error in the measurement of the radius if the percentage error in the volume must be kept within $\pm 3\,\%$.
23. Evaluate the given limit (use L'Hopital's rule when appropriate):

a) $\lim\limits_{x\to 0^+} \frac{\sin x}{x^2}$ b) $\lim\limits_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$ c) $\lim\limits_{x\to \pi^-} (x-\pi)\cot x$ d) $\lim\limits_{x\to 0} (1+2x)^{\frac{-3}{x}}$

e) $\lim\limits_{x\to 0} \left(\frac{1}{x^2} - \frac{\cos(3x)}{x^2}\right)$

Review Exam II solutions

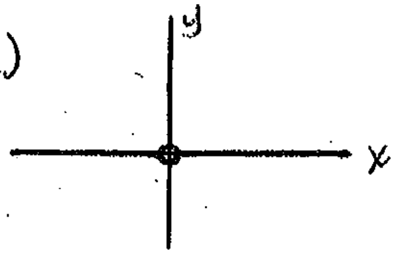
1. $f'(x) = \lim_{h \to 0} \dfrac{\frac{-2}{(x+h)^2} - \left(\frac{-2}{x^2}\right)}{h} = \lim_{h \to 0} \dfrac{1}{h} \dfrac{\left[-2x^2 + 2(x+h)^2\right]}{(x+h)^2 x^2} = \lim_{h \to 0} \dfrac{-2x^2+2x^2+4xh+2h^2}{h(x+h)^2x^2}$

$= \lim_{h \to 0} \dfrac{\cancel{h}}{\cancel{h}} \dfrac{[4x+2h]}{(x+h)^2x^2} = \dfrac{4x}{x^4} = \dfrac{4}{x^3}$

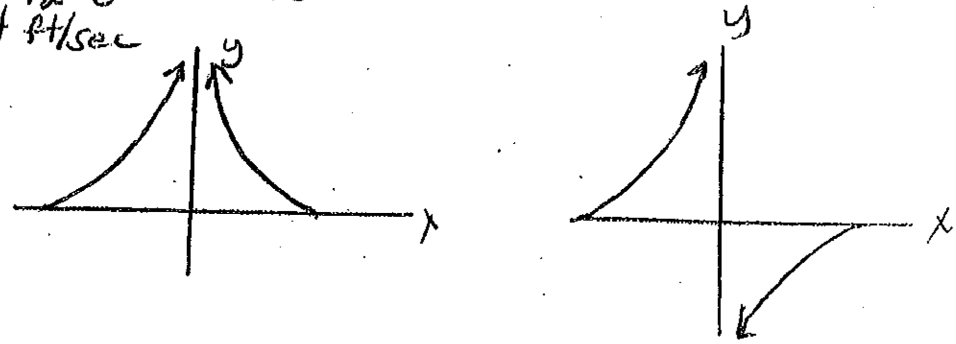
2 a) average velocity $= \dfrac{s(12) - s(0)}{12-0} = \dfrac{648}{12} = 54$ ft/sec.

b) $s'(t) = 9t \quad s'(6) = 54$ ft/sec

3 a)



b)



4. $\dfrac{dy}{dx} = \dfrac{-42}{x^7} - \dfrac{5}{2\sqrt{x}}$

5. $\lim_{x \to \frac{1}{2}^-} f'(x) = \lim_{x \to \frac{1}{2}^-} 3x^2 = \frac{3}{4}$; $\lim_{x \to \frac{1}{2}^+} f'(x) = \lim_{x \to \frac{1}{2}^+} \frac{3}{2}x = \frac{3}{4}$, so $f'(\frac{1}{2}) = \frac{3}{4}$

6. $\dfrac{dy}{dx} = (2x^7 - x^2)\left[\dfrac{(x+1)(1)-(x-1)(1)}{(x+1)^2}\right] + \left(\dfrac{x-1}{x+1}\right)(14x^7 - 2x)$

$\left.\dfrac{dy}{dx}\right|_{x=1} = (1)\left(\frac{2}{4}\right) + 0 = \frac{2}{4}$

7 a) $f'(x) = \dfrac{x^2\cos x - 2x\sin x}{(x^2+\sin x)^2}$

b) $f'(x) = -\sin x + x\csc x\cot x - \csc x$

c) $f'(x) = 0$

8 a) $f'(x) = 6(3x^2+2x-1)^5(6x+2)$

b) $f'(x) = \dfrac{3x^2-2}{2\sqrt{x^3-2x+5}}$

c) $f'(x) = 12x^2\tan^3(x^3)\sec^2(x^3)$

9. $\dfrac{dy}{dx} = \cos x(\cos 3x+1) - 3\sin x\sin(3x+1)$

10. $\dfrac{dy}{dx} = 12\cot^3 x(-\csc^2 x)\Big|_{x=\frac{\pi}{4}} = -24 = m$; $y(\frac{\pi}{4}) = 3$

$y - 3 = -24(x - \frac{\pi}{4})$ or $y = -24x + 3 + 6\pi$

11 a) $y' = \dfrac{-(y^3\sin(xy^3))}{3xy^2\sin(xy^3)+1}$

b) $y' = \dfrac{y(1+\sec y)}{-(3x)(1+\sec y) + xy\sec y\tan y + 4y(1+\sec y)^2}$

12 $y' = \dfrac{-x^2}{y^2}$; $y'' = \dfrac{-2xy^3-2x^4}{y^5} = \dfrac{-2x}{y^5}$

13 a) $\dfrac{dy}{dx} = \dfrac{3x^2-14x}{x^3-7x^2-3}$ b) $\dfrac{dy}{dx} = \dfrac{\sin(\ln x^2)}{x}$

14 $y' = \left[\cot x - \tan x + \dfrac{3\sec^2 x}{\tan x} - \dfrac{1}{2x}\right]\left[\dfrac{\sin x \cos x \tan^3 x}{\sqrt{x}}\right]$

15. $f'(x) = 5x^4 + 24x^2 + 2 > 0$ (Always increasing, so one-to-one

16. $y = f^{-1}(x)$, $x = f(y) = \dfrac{1}{y^2}$ $\dfrac{dx}{dy} = -2y^{-3}$, $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}} = \dfrac{-y^3}{2}$

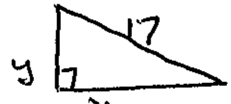
17 a) $\dfrac{dy}{dx} = \dfrac{(-\sin e^x)e^x}{\cos e^x}$ b) $\dfrac{dy}{dx} = \left[\dfrac{2x\ln x}{(x^2+3)} + \dfrac{\ln(x^2+3)}{x}\right](x^2+3)^{\ln x}$

c) $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{4-(x+1)^2}}$ d) $\dfrac{dy}{dx} = \dfrac{5x^4}{|x^5|\sqrt{x^{10}-1}} = \dfrac{5}{|x|\sqrt{x^{10}-1}}$

e) $\dfrac{dy}{dx} = \dfrac{-1}{(\tan^{-1}x)^2}\left(\dfrac{1}{1+x^2}\right)$ f) $\dfrac{dy}{dx} = \dfrac{3x^2(\sin^{-1}x)^2}{\sqrt{1-x^2}} + 2x(\sin^{-1}x)^3$

18. $V = \frac{4}{3}\pi r^3$ Given $\frac{dr}{dt} = -15$ Find $\left.\frac{dV}{dt}\right|_{r=9}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi(9)^2(-15) = -4860\pi$ cm³/min $= -15268.14$ cm³/min

19. 

Find $\left.\frac{dy}{dt}\right|_{y=8}$ - given $\frac{dx}{dt} = 5$; $x^2+y^2 = 17^2$

(when $y=8$, $x=15$)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$; $\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$

$\frac{dy}{dt} = -\frac{15}{8}(5) = -\frac{75}{8}$ ft/s (down)

20 

$V = \frac{1}{3}\pi r^2 h$ Given $\frac{dV}{dt} = 8$, $r = \frac{1}{2}h$, find $\left.\frac{dh}{dt}\right|_{h=6}$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3$; $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$\left.\frac{dh}{dt}\right|_{h=6} = \left.\frac{4}{\pi h^2}\frac{dV}{dt}\right|_{h=6} = \frac{4}{\pi(6)^2}(8) = \frac{8}{9\pi}$ ft/min $\approx .28$ ft/min

21. $f(x) = x^3$; $\quad f'(x) = 3x^2 \quad x_0 = 2, \ \Delta x = -.03$

$(1.97)^3 \approx 2^3 + 12(-.03) = 7.64$

22. $\dfrac{dV}{V} = \dfrac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = 3\dfrac{dr}{r}$; $\quad \dfrac{dV}{V} = \pm .03 \quad 3\dfrac{dr}{r} = \pm .03$

$\dfrac{dr}{r} = \pm .01$ Maximum percentage error in r is $\pm 1\%$

23a) $+\infty$ b) 1 c) 1 d) $\lim\limits_{x\to 0} \ln(1+2x)^{-3/x} = -6$; $\lim\limits_{x\to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$

e) $\frac{9}{2}$.

Review Exam III

1. Find a) the intervals on which f is increasing, b) the intervals on which f is decreasing, c) the open intervals on which f is concave up, d) the open intervals on which f is concave down, e) the x-coordinates of all inflection points/

   i) $f(x) = x^4 - 8x^2 + 16$

   ii) $f(x) = x^{\frac{2}{3}} - x$

2. Locate the critical points and identify which critical points are stationary points for

   $f(x) = 3x^4 + 12x$

3. Use the given derivative to find all critical points of f, and at each critical point determine whether a relative maximum, relative minimum, or neither occurs for

   $f'(x) = \dfrac{x^2 - 7}{\sqrt[3]{x^2 + 4}}$

4. Find the relative extrema using both first and second derivative tests:

   $f(x) = x^4 - 12x^3$

5. Give a graph of the polynomial and label coordinates of the intercepts, stationary points, and inflection points for $p(x) = 2 - x + 2x^2 - x^3$

6. Give a graph of the rational function $f(x) = \dfrac{x^2}{x^2 - 4}$ and label the coordinates of the stationary points and inflection points. Show the horizontal and vertical asymptotes and label them with their equations.

7. Sketch a graph of $f(x) = xe^{-x}$ and identify all relative extrema, inflection points and horizontal asymptote(s).

8. Find the absolute maximum and minimum values of f(x) on the given closed interval and state where those values occur.

   a) $f(x) = 8x - x^2$ over $[0,6]$

   b) $f(x) = \sin x - \cos x$ over $[0, \pi]$

9. Find the absolute maximum and minimum values of f(x) (if any) on the given interval and state where these values occur:

   a) $f(x) = x^4 + 4x$; $(-\infty, +\infty)$

   b) $f(x) = x^3 - 9x + 1$; $(-\infty, +\infty)$

10. The boundary of a field is a right triangle with a straight stream along its hypotenuse and with fences along its other two sides. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.

11. A box with a square base is wider than it is tall. In order to send the box through the U.S. mail, the width of the box and the perimeter of one of the (nonsquare) sides of the box can sum to no more than 108 in. What is the maximum volume for such a box?

12. A closed rectangular container with a square base is to have a volume of 2250 in^3. The material for the top and bottom of the container will cost \$2 per in^2, and the material for the sides will cost \$3 per in^2. Find the dimensions of the container of least cost.
13. A closed cylindrical can is to have a surface area of 96π cm^3. Find the radius and height of the can with maximum volume.
14. A firm determines that x units of its product can be sold daily at p dollars per unit where, p=1000-x; the cost of producing x units per day is C(x) = 3000 + 20x
    a) Find the revenue function b) Find the profit function
    b) Find the maximum profit assuming that production capacity is at most 500 units per day
    e) what price must be charged to obtain maximum profit?
15. A man is floating in a rowboat 1 mile from the straight shoreline of a large lake. A town is located on the shoreline 1 mile from the point on the shoreline closest to the man. He intends to row in a straight line to some point P on the shoreline and then walk the remaining distance to the town. To what point should he row in order to reach his destination in the least time if he can walk 5 mi/h and row 3mi/h?
16. The function $s(t) = t^4 - 4t^2 + 4$, $t \geq 0$ describes the position of a particle moving along a coordinate line, where s is in feet and t is in seconds. a) Find the velocity and acceleration functions. b) Find the position, velocity, speed, and acceleration at time t=1. c) At what times is the particle stopped? d)When is the particle speeding up? Slowing down? e) Find the total distance traveled by the particle from time t=0 to time t=5.
17. Verify that $f(x) = x - \frac{1}{x}$ over [3,4] satisifies the hypotheses of the Mean Value Theorem on the given interval, and find all values of c in the interval that satisfy the conclusion of the theorem.
18. Use the Mean Value Theorem to prove that if f is differentiable on an open interval, and if $|f'(x)| \geq M$ for all values of x in the interval, then $|f(x)-f(y)| \geq M|x-y|$ for all values x and y in the interval.
19. Evaluate the integrals:
    a) $\int (\frac{10}{y^{\frac{3}{4}}} - \sqrt[3]{y} + \frac{4}{\sqrt{y}})\,dy$ b) $\int [\phi + \frac{2}{\sin^2\phi}]d\phi$ c) $\int \frac{\sec x + \cos x}{2\cos x}dx$
    d) Solve $\frac{dy}{dt} = \sec^2 t - 5\sin t$; $y(\frac{\pi}{4}) = 1$
20. Solve $\frac{dy}{dt} = \frac{1}{t}$, y(-1)=5 for y(t)
21. A particle moves along an x-axis with position function s=s(t), and velocity function $v(t)=3e^t$; s(1)=0. Find s(t).

22. Evaluate the integrals:

a) $\int \frac{x^3}{\sqrt{x^4+6}}dx$ b) $\int \frac{1}{1+16x^2}dx$ c) $\int \frac{x^2+1}{\sqrt[3]{x^3+3x}}\,dx$ d) $\int x^3 e^{x^4}\,dx$ e) $\int \frac{x^2+3}{x^3+x}dx$

f) $\int \frac{dx}{\sqrt{9-x^2}}$ g) $\int \frac{dx}{x^2+16}$

23. Sketch the curve by eliminating the parameter:

a) $x=\sqrt{t}$, $y=2t+4$

b) $x=\sec^2 t$, $y=3\cos^2 t$ $(0 \le t \le \pi/2)$

24. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ without eliminating the parameter:

a) $X=\frac{1}{2}t^2+1$, $y=\frac{1}{3}t^3-t$; $t=2$

b) $X=\cos\phi$, $y=3\sin\phi$, $\phi=\frac{5\pi}{6}$

Review Exam III - Solutions

1 i) $f'(x) = 4x(x-2)(x+2)$ a) $[-2,0]$, $[2,+\infty)$ b) $(-\infty,-2]$, $[0,2]$

$f''(x) = 12\left(x-\frac{2}{\sqrt3}\right)\left(x+\frac{2}{\sqrt3}\right)$ c) $\left(-\infty,-\frac{2}{\sqrt3}\right)$, $\left(\frac{2}{\sqrt3},+\infty\right)$ d) $\left(-\frac{2}{\sqrt3},\frac{2}{\sqrt3}\right)$

e) $\left(-\frac{2}{\sqrt3},\frac{2}{\sqrt3}\right)$

ii) $f'(x) = \frac{2}{3}x^{-\frac{1}{3}} - 1$ a) $\left[0,\frac{8}{27}\right]$ b) $(-\infty,0]$, $\left[\frac{8}{27},+\infty\right)$

$f''(x) = -\frac{2}{9}x^{-4/3}$ c) Nowhere d) $(-\infty,0)$, $(0,+\infty)$ e) None

2. $f'(x) = 12x^3 + 12(x+1)(x^2-x+1)$ so $x=-1$ stationary point

3. C.P. $x = \pm\sqrt7$ f' $\underset{-\sqrt7}{+ \mid} \; - \; \underset{\sqrt7}{\mid +}$ At $x=-\sqrt7$ rel max

At $x=\sqrt7$ rel min

4. $f'(x) = 4x^3 - 36x^2$, critical points at $x = 0, 9$

f' $\underset{0}{- \mid} \; - \; \underset{9}{\mid +}$ Rel min at $x=9$ No rel max

$f''(x) = 12x^2 - 72x$ $f''(0) = 0$ Inconclusive, $f''(9) > 0$ so rel min at $x=9$

5. $p(0) = 2$, $p(2) = 0$ $p'(x) = -1 + 4x - 3x^2$, C.P. $x = \frac{1}{3}, 1$ f' $\underset{\frac13}{- \mid} \; + \; \underset{1}{\mid -}$

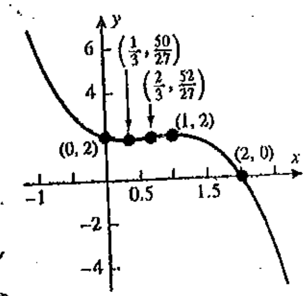
f dec $\left(-\infty,\frac13\right] \cup [1,+\infty)$; f inc $\left[\frac13, 1\right]$

$\left(\frac13, \frac{50}{27}\right)$ rel min; $(1,2)$ rel max

$p''(x) = 4 - 6x = 0$ $x = \frac23$ f'' $+ \underset{\frac23}{\mid} -$

$\left(\frac23, \frac{52}{27}\right)$ inflection point

f concave up $\left(-\infty, \frac23\right)$; f concave down $\left(\frac23, +\infty\right)$

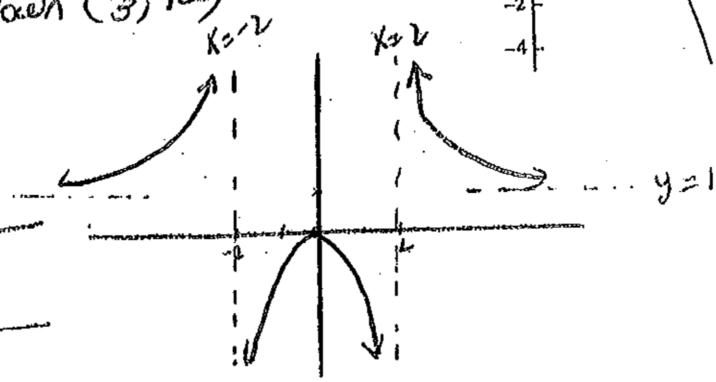


6 Vertical asymptotes $x = \pm 2$

Horizontal asymptotes $y = 1$

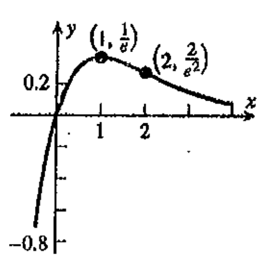
$f'(x) = \frac{-8x}{(x^2-4)^2}$ f' $+ \underset{-2}{\}} + \underset{0}{\mid} - \underset{2}{\}} -$

$f''(x) = \frac{16x^2+32}{(x^2-4)^3}$ f'' $+ \underset{-2}{\}} - \underset{2}{\}} +$



7. (a) $\lim_{x\to+\infty} f(x) = 0$, $\lim_{x\to-\infty} f(x) = -\infty$

(b) $f'(x) = (1-x)e^{-x}$, $f''(x) = (x-2)e^{-x}$
critical point at $x = 1$;
relative maximum at $x = 1$
point of inflection at $x = 2$
horizontal asymptote $y = 0$ as $x \to +\infty$

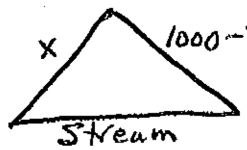


8a) $f'(x)=8-2x$, $f'(x)=0$ at $x=4$; $f(0)=0$, $f(4)=16$, $f(6)=12$
Max $(4,16)$ Min $(0,0)$

b) $f'(x)=\cos x+\sin x$, $f'(x)=0$ at $x=\frac{3\pi}{4}$, $f(0)=-1$, $f(\frac{3\pi}{4})=\sqrt{2}$, $f(\pi)=1$
Max $(\frac{3\pi}{4},\sqrt{2})$ Min $(0,-1)$

9 a) $f'(x)=4(x^3+1)$ c.p. $x=-1$, Min $(-1,-3)$; No Max

b) $f'(x)=3x^2-9$ c.p. $x=\pm\frac{1}{\sqrt{3}}$ $\quad + \quad -\sqrt{3} \quad - \quad \sqrt{3} \quad +$ No MAX, NO MIN

10. 

$A(x)=\frac{1}{2}x(1000-x)=500x-\frac{1}{2}x^2$ over $[0,1000]$
$A'(x)=500-x=0$, $x=500$

x	$A(x)$	
0	0	
500	125,000	MAX
1000	0	

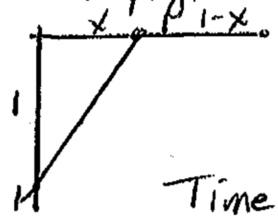
MAX when both sides 500'

11 Dimensions x, y, $x \geq y$, $x+2(x+y)\leq 108$
$V=x^2y$ $\quad y=\frac{108-3x}{2}$; $V(x)=x^2(\frac{108-3x}{2})=54x^2-\frac{3}{2}x^3$, $x>0$
$V'(x)=108x-\frac{9}{2}x^2=0$ $\quad x=24$
$V''(x)=108-\frac{9}{4}x$, $V''(24)<0$ so MAX Volume, $x=24$, $y=18$

12. Let x= edge at base, y= height, $V=x^2y=2250$, $x>0$
$y=\frac{2250}{x^2}$; $\quad$ Cost $=2(2x^2)+3(4xy)=4x^2+12xy$
$C(x)=4x^2+12x(\frac{2250}{x^2})=4x^2+\frac{27000}{x}$; $C'(x)=8x-\frac{27000}{x^2}=0$, $x=15$
$C''(x)=8+\frac{54000}{x}$, $C''(15)>0$ so MIN when $x=15$, $y=10$

13. Surface area $=2\pi r^2+2\pi rh=96\pi$, $h=\frac{48-r^2}{r}$
Volume, $V=\pi r^2h=48\pi r-\pi r^3$; $V'(r)=48\pi-3\pi r^2=0$
$r^2=16$, $r=4$; $V''(r)=-6\pi r$, $V''(4)<0$ so Abs Max, $r=4$, $h=8$

14 a) $R(x)=(1000-x)x$; b) $P(x)=1000x-x^2-(3000+20x)=-3000+980x-x^2$
c) $P'(x)=980-2x=0$, $x=490$, $P''(x)=-2$, $P''(490)<0$ so MAX profit when $x=490$; d) $P(490)=\$237,100$; e) $p=1000-490=\$510$

15 

a) Time $T(x)=\frac{\sqrt{1+x^2}}{3}+\frac{1-x}{5}$ $\quad 0\leq x\leq 1$
$T'(x)=\frac{x}{3\sqrt{x^2+1}}-\frac{1}{5}=0$; $x=\frac{3}{4}$

x	$T(x)$	
0	$8/15$	
$\frac{3}{4}$	$7/15$	← MIN
1	$\sqrt{2}/3$	

Time minimum when $x=\frac{3}{4}$.

16 a) $V(t)=4t^3-8t$, $a(t)=12t^2-8$
b) $S(1)=1$ ft, $V(1)=-4$ ft/s, Speed $=4$ ft/s, $a(1)=4$ ft/s^2
c) $V=0$ at $t=0, \sqrt{2}$
d) Speeding up $0<t<\frac{\sqrt{6}}{3}$ and $t>\sqrt{2}$, slowing down $\frac{\sqrt{6}}{3}<t<\sqrt{2}$
e) Total distance $=|S(\sqrt{2})-S(0)|+|S(5)-S(\sqrt{2})|=|0-4|+|529-0|=533$ ft.

17. $f(4)=\frac{15}{4}$, $f(3)=\frac{8}{3}$ Solve $f'(c)=\dfrac{(\frac{15}{4}-\frac{8}{3})}{4-3}=\frac{13}{12}$

$f'(x)=1+\frac{1}{x^2}$, $f'(c)=1+\frac{1}{c^2}=\frac{13}{12}$, $c^2=12$, $c=\pm 2\sqrt{3}$ so $c=2\sqrt{3}$

18. If $x,y\in I$, $x<y$ then for some c in I, $\frac{f(y)-f(x)}{y-x}=f'(c)$

so $|f(x)-f(y)|=|f'(c)|\,|x-y|\ge H\,|x-y|$ for all x,y in I.

19 a) $40y^{\frac{1}{4}}-\frac{3}{4}y^{4/3}+8\sqrt{y}+C$ b) $\frac{\varphi^2}{2}-2\cot\varphi+C$ c) $\frac{1}{2}\tan x+\frac{1}{2}x+C$

d) $y(t)=\tan t+\cos t+C$ $y(\frac{\pi}{4})=1$, $C=-\frac{\sqrt{2}}{2}$

$y(t)=\tan t+\cos t-\frac{\sqrt{2}}{2}$

20. $y=\ln|t|+5$

21. $s(t)=\int 3e^t\,dt=3e^t+C$, $s(1)=3e+C=0$, $C=-3e$

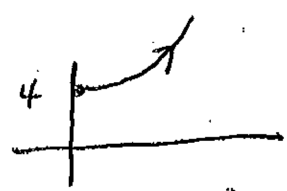
so $s(t)=3e^t-3e$.

22 a) $\frac{1}{2}\sqrt{x^4+6}+C$ b) $\frac{1}{4}\tan^{-1}(4x)+C$ c) $\frac{1}{3}\sqrt[3]{(x^3+3x)^2}+C$

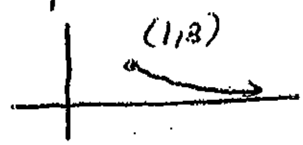
d) $\frac{1}{4}e^{x^4}+C$ e) $\frac{1}{3}\ln|x^3+x|+C$ f) $\sin^{-1}(\frac{x}{3})+C$

g) $\frac{1}{4}\tan^{-1}(\frac{x}{4})+C$

23 a) $t=x^2$ $y=2x^2+4$
$(x\ge 0)$



b) $y=\frac{3}{x}$, $x\ge 1$, $y>0$



24 a) $\frac{dy}{dx}=\frac{t^2-1}{t}=t-\frac{1}{t}$ $\frac{d^2y}{dx^2}=\dfrac{(1+\frac{1}{t^2})}{t}=\frac{t^2+1}{t^3}$

$\left.\frac{dy}{dx}\right|_{t=2}=\frac{3}{2}$ $\left.\frac{d^2y}{dx^2}\right|_{t=2}=\frac{5}{8}$

b) $\frac{dy}{dx}=\frac{3\cos\varphi}{-\sin\varphi}=-3\cot\varphi$, $\frac{d^2y}{dx^2}=\frac{-3(-\csc^2\varphi)}{-\sin\varphi}=-3\csc^3\varphi$

$\left.\frac{dy}{dx}\right|_{\varphi=\frac{5\pi}{6}}=3\sqrt{3}$, $\left.\frac{d^2y}{dx^2}\right|_{\varphi=\frac{5\pi}{6}}=-24$