Suggested Answers for the Test

Finished by Yi Ding

March 28, 2010
First way, considering the two firms separately

\[ MC_A = 4Q_A = 10 - 2(Q_A + Q_B) = MR \]
\[ MC_B = 2 + 2Q_B = 10 - 2(Q_A + Q_B) = MR \]

Or considering from the point of the monopolist, the monopolist want to solve the following maximum problem:

\[ \max_{Q_A, Q_B} \pi = PQ - C_A(Q_A) - C_B(Q_B) \]
\[ = (10 - Q_A - Q_B)(Q_A + Q_B) - C_A(Q_A) - C_B(Q_B) \]

Where \( C_i(Q_i) \ (i = A, B) \) is the cost function. From the F.O.C, we get

\[
\begin{align*}
\frac{\partial \pi}{\partial Q_A} &= 10 - 2(Q_A + Q_B) - MC_A = 0 \\
\frac{\partial \pi}{\partial Q_B} &= 10 - 2(Q_A + Q_B) - MC_B = 0
\end{align*}
\]

Which will yield the same results as the first way.
For any other way, either Player A or Player B will unilaterally deviate.

11 Answers:

11.1 Marginal Revenue Function:

\[ R = PQ = Q(20 - Q) = 20Q - Q^2 \Rightarrow MR = 20 - 2Q \]

11.2 Profit function of A:

\[ \pi_A = PQ_A - C(Q_A) = (20 - Q_A - Q_B)Q_A - 8Q_A = -Q_A^2 + (12 - Q_B)Q_A \]

11.3 The best response function for A would reflect how much A would like to produce given each \( Q_B \), solving the following maximum problem:

\[ \max_{Q_A} \pi_A = -Q_A^2 + (12 - Q_B)Q_A \]

Considering the F.O.C

\[ \frac{\partial \pi_A}{\partial Q_A} = -2Q_A + 12 - Q_B = 0 \Rightarrow Q_A^* = 6 - \frac{1}{2}Q_B \]

Which yields the best response function will be

\[ BR_A(Q_B) = 6 - \frac{1}{2}Q_B \]
11.4 Cournot competition equilibrium comes when each firm make the best response to the other. By considering these two firms are identical, it is easily to write the firm B’s best response function, $BR_B(Q_B) = 6 - \frac{1}{2}Q_A$, at the equilibrium, suppose they produce $Q_A^*$, $Q_B^*$ respectively, so

$$\begin{align*}
Q_A^* &= BR_A(Q_B) = 6 - \frac{1}{2}Q_B^* \\
Q_B^* &= BR_B(Q_A) = 6 - \frac{1}{2}Q_A^*
\end{align*}$$

Solving this linear system will yield

$$\begin{align*}
Q_A^* &= 4 \\
Q_B^* &= 4
\end{align*}$$

11.5 The market equilibrium price will be

$$P_e = 20 - Q_A^* - Q_B^* = 20 - 4 - 4 = 12$$

11.6 The profit for each firm will be identical, it will be

$$\pi_i = -Q_i^2 + (12 - Q_{-i})Q_i = 16, (i = A, B; -i = B, A)$$

11.7 If they perfectly collude, they will operate according monopoly and share the profit. When they perfectly collude,

$$MC = 8 = 20 - 2Q = MR$$

which will reach $Q = 6$, then each will produce $Q_A = Q_B = 3$, the profit for each firm will be

$$\pi_i = (20 - Q)Q_i - 8Q_i = (20 - 6) * 3 - 8 * 3 = 18, (i = A, B)$$
11.8 By the best response function, take firm A as an example, when they reach the perfect collusion, we have $Q_A = Q_B = 3$, however, given $Q_B = 3$, the firm A’s best response would be

$$BR_A(3) = 6 - \frac{1}{2} \times 3 = 4.5 > 3$$

so the firm A will have the incentive to unilaterally deviate from the monopoly collusion. The same is the firm B.